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## MATRIX METHODS FOR DETERMINING THE LONGITUDINAL-STABILITY DERIVATIVES OF AN AIRPLANE FROM TRANSIENT FLIGHT DATA

By James J. Donegan

Langley Aeronautical Laboratory
Langley Field, Va.

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## SUMMARY

Three matrix methods are presented for determining the longitudinal-stability derivatives from transient flight data. One method, which requires four measurements in time-history form and utilizes the incremental tail load to separate the pitching-moment derivatives $C_{m_{\dot{\theta}}}$ and $C_{m_{\dot{\alpha}}}$, permits the computation of all the longitudinal-stability derivatives. A second method requires three measurements and one supplemental assumption, namely $\frac{C_{m_{\dot{\alpha}}}}{C_{m_{\dot{\theta}}}} = \text{Constant}$. This method gives the most information for the least amount of work. The third method requires two measurements and two supplemental assumptions, namely $\frac{C_{m_{\dot{\alpha}}}}{C_{m_{\dot{\theta}}}} = \text{Constant}$ and $C_{m_\delta} = \frac{x_t}{\bar{c}} C_{L_\delta}$ (where $C_{m_\delta}$ and $C_{L_\delta}$ are the elevator-effectiveness derivatives, $x_t$ is the tail length, and $\bar{c}$ is the mean aerodynamic chord). An inspection of the results obtained for the various methods shows the scatter which is typical of this type of analysis of flight data.

## INTRODUCTION

The determination of the longitudinal-stability derivatives from flight data is a relatively difficult task because the wind-tunnel technique of permitting only one variable to change at a time, while constraining all the rest of the variables, cannot always be used. It is in the analysis of such flight-test data that matrix techniques employing the equations of motion seem to be particularly useful.

Currently, much work is being carried out on the determination of stability derivatives directly from flight data but as yet this work is

still in the preliminary stages. The matrix methods for the determination of stability derivatives from transient flight data that are developed herein are an addition to this work. The previous work done on the determination of longitudinal-stability derivatives is extensive, and no attempt is made to summarize it since this summarization has been adequately done in reference 1.

In the present paper three methods are developed and presented for determining the longitudinal-stability derivatives from transient flight data. In these methods the expressions for some of the stability derivatives are in the form generally used in stability calculations. The first method requires the combination of four measurements in time-history form, two of which must be incremental elevator deflection and incremental tail load and the other two measurements can be chosen from a possible three, namely incremental load factor, pitching velocity, and angle of attack. The method demonstrates the use of the tail load to separate the pitching-moment derivatives $C_{m_{\dot{\theta}}}$ and $C_{m_{\dot{\alpha}}}$ and determine the downwash derivative $\partial\epsilon/\partial\alpha$.

The second method, which is more restricted, requires a combination of three measurements (in time-history form), one of which must be incremental elevator deflection and the other two measurements can be chosen from a possible three, namely incremental load factor, pitching velocity, and angle of attack. This method also requires one supplementary assumption, namely $C_{m_{\dot{\alpha}}} = \lambda C_{m_{\dot{\theta}}}$, where $\lambda$ is a constant.

The third method uses a combination of two measurements (in time-history form), one of which must be incremental elevator deflection and the other one may be chosen from incremental load factor, pitching velocity, angle of attack, and so forth. The method also requires two supplementary assumptions, namely $C_{m_{\dot{\alpha}}} = \lambda C_{m_{\dot{\theta}}}$ and $C_{m_\delta} = \frac{x_t}{\bar{c}} C_{L_\delta}$ (where $C_{m_\delta}$ and $C_{L_\delta}$ are the elevator-effectiveness derivatives, $x_t$ is the tail length, and $\bar{c}$ is the mean aerodynamic chord). By using a modification of the third method, it is shown that considerable information can be obtained from a single time history.

The methods are demonstrated by applying them to flight data obtained from tests of a medium jet bomber, and a comparison of the derivatives obtained by the various methods gives an indication of the accuracy which can be expected from data analysis by matrix techniques based on the longitudinal equations of motion.

## SYMBOLS

| | |
|---|---|
| $b$ | wing span, ft |
| $\bar{c}$ | mean aerodynamic chord |
| $C_1,C_2$ | constants defined in appendix E |
| $C_L$ | lift coefficient, $L/qS$ |
| $C_{L_\alpha}$ | rate of change of airplane lift coefficient with angle of attack per radian; see appendix E |
| $C_{L_\delta}$ | rate of change of lift coefficient with elevator deflection per radian; see appendix E |
| $C_{L_{\dot{\alpha}}}$ | rate of change of lift coefficient with $\dot{\alpha}$ per radian; see appendix E |
| $C_{L_{\dot{\theta}}}$ | rate of change of lift coefficient with pitching velocity per radian; see appendix E |
| $C_m$ | pitching-moment coefficient of airplane, $M/qS\bar{c}$ |
| $C_{m_\alpha}$ | rate of change of pitching-moment coefficient with angle of attack per radian; see appendix E |
| $C_{m_\delta}$ | rate of change of pitching-moment coefficient with elevator deflection per radian; see appendix E |
| $C_{m_{\dot{\theta}}}$ | rate of change of pitching-moment coefficient with pitching velocity per radian; see appendix E |
| $C_{m_{\dot{\alpha}}}$ | rate of change of pitching-moment coefficient with $\dot{\alpha}$ per radian; see appendix E |
| $C_{m_t}$ | pitching-moment coefficient of horizontal tail surface, $M_t/q_tS_t\bar{c}_t$ |
| $g$ | acceleration due to gravity, ft/sec/sec |
| $I$ | airplane moment of inertia, slug-ft$^2$ |
| $k_y$ | airplane radius of gyration about pitching axis, ft |
| $L$ | lift, lb |

| | |
|---|---|
| $m$ | airplane mass, $W/g$, slugs |
| $M$ | pitching moment of airplane |
| $n$ | airplane load factor |
| $q$ | dynamic pressure, $\frac{\rho V^2}{2}$, lb/sq ft |
| $S$ | wing area, sq ft |
| $S_t$ | horizontal-tail area |
| $t$ | time, sec |
| $V$ | true velocity, ft/sec |
| $W$ | airplane weight, lb |
| $x_t$ | length from center of gravity of airplane to aerodynamic center of tail (negative for conventional airplanes), ft |
| $K_1,K_2,K_5,K_6$ | coefficients of transfer function relating $\theta$ and $\delta$; see appendix E |
| $\alpha$ | wing angle of attack, radians |
| $\alpha_t$ | tail angle of attack, radians |
| $\gamma$ | flight-path angle, radians |
| $\theta$ | angle of pitch, $\alpha + \gamma$ |
| $\delta$ | elevator deflection, radians |
| $\epsilon$ | downwash angle, radians |
| $\lambda = \frac{C_{m_{\dot{\alpha}}}}{C_{m_{\dot{\theta}}}}$ | |
| $\eta_t$ | tail efficiency factor, $q_t/q$ |
| $\rho$ | mass density of air, slugs/cu ft |
| $\tau$ | dummy variable of integration |

Matrix notation:

| | |
|---|---|
| $\| \ \|$ | rectangular matrix |
| $[\ ]$ | square matrix |
| $\{\ \}$ | column matrix |
| $\|C\|$ | integrating matrix (see table I) |
| $\|B\|, \|D\|, \|E\|$ | rectangular matrices defined in appendix E |

Subscripts:

| | |
|---|---|
| i | denotes row elements in matrix |
| t | tail |
| WB | wing-body combination |

For sign conventions used, see figure 1.

A dot over a symbol denotes the first derivative with respect to time, and two dots over a symbol denote the second derivative with respect to time.

The symbol $\Delta$ refers to an incremental value. Intermediate variables such as $\Delta\mu$, $\Delta\xi$, $\Delta\sigma$, $\Delta\varphi$, and $\Delta\psi$ and the constant $K_{10}$ are defined in appendix E.

## OUTLINE OF METHODS

The three methods are based on the longitudinal equations of motion for horizontal flight and use matrix methods to analyze time histories of measured quantities. The equations of motion used in each of these methods are expressed in the form

$$\frac{W}{qS} \Delta n = C_{L_\alpha} \Delta\alpha + C_{L_{\dot{\theta}}} \dot{\theta} + C_{L_{\dot{\alpha}}} \dot{\alpha} + C_{L_\delta} \Delta\delta \tag{1}$$

$$\frac{I}{qS\bar{c}} \ddot{\theta} = C_{m_\alpha} \Delta\alpha + C_{m_{\dot{\alpha}}} \dot{\alpha} + C_{m_{\dot{\theta}}} \dot{\theta} + C_{m_\delta} \Delta\delta \tag{2}$$

These equations apply to a rigid airplane and are based on the usual assumptions of linearity, small angles, and no loss in airspeed during the maneuver. The equations are further restricted to the range in which the variation of the derivatives is linear and also to conventional wing-tail configurations in which the major contribution to damping in pitch is due to the horizontal tail. All the variables are given in incremental form measured from a steady-flight trim condition.

As indicated in reference 2, the four values $\Delta\alpha$, $\dot{\alpha}$, $\dot{\theta}$, and $\Delta\delta$ in equations (1) and (2) are linearly dependent; therefore, if four simultaneous equations are formed to determine either the force or moment derivatives, they cannot be solved uniquely for the unknowns.

For purposes of analysis the moment equation (2) is integrated once and expressed in the form

$$\frac{I}{qS\bar{c}} \dot{\theta} = C_{m_\alpha} \int_0^t \Delta\alpha \, dt + C_{m_{\dot{\alpha}}} \Delta\alpha + C_{m_{\dot{\theta}}} \Delta\theta + C_{m_\delta} \int_0^t \Delta\delta \, dt \tag{3}$$

This form permits the use of numerical integrating methods that are more desirable than numerical differentiating schemes when applied to flight data. Integrations of the variables are performed by use of an integrating matrix $\|C\|$ derived in reference 3 and given in table I herein. For instance, a time history of $\int_0^t \Delta n \, dt$ may be obtained from a time history of $\Delta n$ as follows:

$$\left\{\int_0^t \Delta n \, dt\right\} = \|C\| \left\{\Delta n_i\right\} \tag{4}$$

The integrating matrix $\|C\|$ given in table I may be used for any time interval $\Delta t$; most of the computations of this paper are based on a time inverval of $\Delta t = 0.1$ second. This interval may be too large in some cases, and if greater accuracy is desired, a shorter time interval may be chosen.

The essential differences in each of the methods are in the number of quantities to be measured. Method A requires four basic measurements in time-history form to determine all the derivatives. Method B requires three measurements and one supplemental assumption, namely $C_{m_{\dot{\alpha}}} = \lambda C_{m_{\dot{\theta}}}$. Method C requires two measurements and two supplemental assumptions,

namely $C_{m_{\dot{\alpha}}} = \lambda C_{m_{\dot{\theta}}}$ and $C_{m_{\delta}} = \frac{x_t}{\bar{c}} C_{L_{\delta}}$. All measurements of flight data used are time histories of incremental values measured from a trimmed level-flight initial position. The development of the equations for each method is covered in appendixes A to C; in the body of the report the methods are outlined by stating the pertinent equations in the order of computation. Since these computations make extensive use of least-squares procedures and are greatly facilitated by the use of matrix algebra, most of the equations are given in matrix form.

## Method A

Of the four basic measurements required with method A, two must be incremental elevator angle and incremental tail load and two other measurements can be chosen from a possible three, namely incremental load factor, pitching velocity, and angle of attack. In this paper, incremental load factor and pitching velocity are used.

The procedure of computation with method A (see appendix A for development) is as follows:

(1) Compute a time history of rate of change of angle of attack $\dot{\alpha}$ from

$$\dot{\alpha} = \dot{\theta} - \frac{g}{V} \Delta n \tag{5}$$

or

$$\left\{\dot{\alpha}_i\right\} = \left\{\dot{\theta}_i\right\} - \frac{g}{V} \left\{\Delta n_i\right\}$$

(2) Calculate time histories of $\Delta\alpha$, $\Delta\theta$, $\int_0^t \Delta\theta \, dt$, $\int_0^t \Delta\delta \, dt$, and $\int_0^t \int_0^\tau \Delta\delta \, d\tau \, dt$ by using the integrating matrix $\|C\|$ and the time histories of $\dot{\alpha}$, $\dot{\theta}$, and $\Delta\delta$; for example,

$$\left\{\Delta\alpha_i\right\} = \|C\| \left\{\dot{\alpha}_i\right\} \tag{6}$$

(3) Determine $C_{L_\alpha}$ and $C_{L_\delta}$ by least squares from the relation

$$C_{L_\alpha} \Delta\alpha + C_{L_\delta} \Delta\delta = \frac{W}{qS} \Delta n \qquad (7)$$

or

$$\|B\| \begin{Bmatrix} C_{L_\alpha} \\ C_{L_\delta} \end{Bmatrix} = \frac{W}{qS} \left\{\Delta n_i\right\}$$

(4) Compute the coefficients $K_1$, $K_2$, $K_5$, and $K_6$ of the transfer function relating pitching velocity and elevator deflection by the use of the method of reference 4 and the equation

$$K_1 \Delta\theta + K_2 \int_0^t \Delta\theta \, dt - K_5 \int_0^t \Delta\delta \, dt - K_6 \int_0^t \int_0^\tau \Delta\delta \, d\tau \, dt = -\dot{\theta} \qquad (8)$$

where the measured values of pitching velocity and elevator deflection are used.

(5) Determine $K_{10}$ from the relation

$$K_{10} = \frac{I}{\bar{c}Vm} C_{L_\alpha} - \frac{I}{qS\bar{c}} K_1 \qquad (9)$$

by using the results of steps (3) and (4).

(6) Calculate time histories of the intermediate quantities $\Delta\varphi$ and $\Delta\mu$ by using the expressions

$$\Delta\varphi = \frac{\bar{c}}{x_t}\left[\frac{g}{V} \Delta n - \frac{V}{x_t}\left(\sqrt{\eta_t} + 1\right)\Delta\alpha\right] \qquad (10)$$

or

$$\left\{\Delta\varphi_i\right\} = \frac{\bar{c}g}{x_t V} \left\{\Delta n_i\right\} - \frac{\bar{c}V}{{x_t}^2}\left(\sqrt{\eta_t} + 1\right) \left\{\Delta\alpha_i\right\}$$

and

$$\Delta\mu = \frac{\Delta L_t}{qS} - \frac{\bar{c}V}{x_t^2} K_{10}\,\Delta\alpha - \frac{\bar{c}}{x_t} K_{10}\dot{\alpha} - C_{L_\delta}\,\Delta\delta \tag{11}$$

or

$$\left\{\Delta\mu_i\right\} = \frac{1}{qS}\left\{\Delta L_{t_i}\right\} - \frac{\bar{c}V}{x_t^2} K_{10}\left\{\Delta\alpha_i\right\} - \frac{\bar{c}}{x_t} K_{10}\left\{\dot{\alpha}_i\right\} - C_{L_\delta}\left\{\Delta\delta_i\right\}$$

(7) Compute $C_{m_{\dot{\theta}}}$ by least squares from the relation

$$C_{m_{\dot{\theta}}}\,\Delta\varphi = \Delta\mu \tag{12}$$

or

$$C_{m_{\dot{\theta}}}\left\{\Delta\varphi_i\right\} = \left\{\Delta\mu_i\right\}$$

(8) Determine $C_{m_{\dot{\alpha}}}$ from the equation

$$C_{m_{\dot{\alpha}}} = K_{10} - C_{m_{\dot{\theta}}} \tag{13}$$

(9) Calculate the time history of the intermediate quantity $\Delta\sigma$ from

$$\Delta\sigma = \frac{I}{qS\bar{c}}\,\dot{\theta} - C_{m_{\dot{\alpha}}}\,\Delta\alpha - C_{m_{\dot{\theta}}}\,\Delta\theta \tag{14}$$

or

$$\left\{\Delta\sigma_i\right\} = \frac{I}{qS\bar{c}}\left\{\dot{\theta}_i\right\} - C_{m_{\dot{\alpha}}}\left\{\Delta\alpha_i\right\} - C_{m_{\dot{\theta}}}\left\{\Delta\theta_i\right\}$$

(10) Compute $C_{m_\alpha}$ and $C_{m_\delta}$ by least squares by using the relation

$$C_{m_\alpha} \int_0^t \Delta\alpha \; dt + C_{m_\delta} \int_0^t \Delta\delta \; dt = \Delta\sigma \qquad (15)$$

or

$$\|D\| \begin{Bmatrix} C_{m_\alpha} \\ C_{m_\delta} \end{Bmatrix} = \left\{\Delta\sigma_i\right\}$$

(11) Calculate $C_{L_{\dot\theta}}$ and $C_{L_{\dot\alpha}}$ from the following definitions:

$$C_{L_{\dot\theta}} = \frac{\bar{c}}{x_t} C_{m_{\dot\theta}} \qquad (16)$$

and

$$C_{L_{\dot\alpha}} = \frac{\bar{c}}{x_t} C_{m_{\dot\alpha}} \qquad (17)$$

(12) Determine the time history of the intermediate quantity $\Delta\psi$ from the equation

$$\Delta\psi = \frac{W}{qS} \Delta n - C_{L_{\dot\theta}}\dot\theta - C_{L_{\dot\alpha}}\dot\alpha \qquad (18)$$

or

$$\left\{\Delta\psi_i\right\} = \frac{W}{qS}\left\{\Delta n_i\right\} - C_{L_{\dot\theta}}\left\{\dot\theta_i\right\} - C_{L_{\dot\alpha}}\left\{\dot\alpha_i\right\}$$

(13) Compute the refined values of $C_{L_\alpha}$ and $C_{L_\delta}$ by least squares by using the relation

$$C_{L_\alpha} \Delta\alpha + C_{L_\delta} \Delta\delta = \Delta\psi \qquad (19)$$

or

$$\|B\| \begin{Bmatrix} C_{L_\alpha} \\ C_{L_\delta} \end{Bmatrix} = \left\{ \Delta\psi_i \right\}$$

These values now include the effects of the $C_{L_{\dot{\theta}}}$ and $C_{L_{\dot{\alpha}}}$ terms in the force equation.

(14) Method A can now be iterated to obtain better values of the derivatives by starting the process over at step (5) with the improved $C_{L_\alpha}$ and $C_{L_\delta}$ values from step (13) and following the procedure again. The iteration converges rapidly.

(15) The derivatives $\left(\frac{\partial C_L}{\partial \alpha}\right)_t$, $\frac{\partial \epsilon}{\partial \alpha}$, and $\frac{\partial C_{L_t}}{\partial \delta}$ are found from

$$\left(\frac{\partial C_L}{\partial \alpha}\right)_t = - \frac{S\bar{c}V}{S_t {x_t}^2 \sqrt{\eta_t}} C_{m_{\dot{\theta}}} \tag{20}$$

$$\frac{\partial \epsilon}{\partial \alpha} = \frac{1}{\sqrt{\eta_t}} \frac{C_{m_{\dot{\alpha}}}}{C_{m_{\dot{\theta}}}} \tag{21}$$

$$\frac{\partial C_{L_t}}{\partial \delta} = \frac{S}{\eta_t S_t} C_{L_\delta} \tag{22}$$

This procedure shows that the derivatives $C_{L_\alpha}$, $C_{L_{\dot{\theta}}}$, $C_{L_{\dot{\alpha}}}$, $C_{L_\delta}$, $C_{m_\alpha}$, $C_{m_{\dot{\theta}}}$, $C_{m_{\dot{\alpha}}}$, $C_{m_\delta}$, $\left(\frac{\partial C_L}{\partial \alpha}\right)_t$, $\frac{\partial \epsilon}{\partial \alpha}$, and $\frac{\partial C_{L_t}}{\partial \delta}$ may be determined by numerical operations on four time histories of measured flight data and through the use of the theoretical relationships given as equations (16), (17), (20), (21), and (22).

## Method B

Three basic measurements are used in method B, one of which must be incremental elevator angle and the other two measurements can be chosen from a possible three, namely incremental load factor, pitching velocity, and angle of attack. In this paper, incremental load factor and pitching velocity were used.

In lieu of the fourth measurement, the supplemental assumption is made that

$$C_{m_{\dot{\alpha}}} = \lambda C_{m_{\dot{\theta}}} \tag{23}$$

If a value of $\lambda$ is not known in advance, a first approximation (see ref. 5) is $\lambda = \frac{1}{2}$. Although the assumption $\lambda = \frac{1}{2}$ imposes a restriction on the generality of the method, it appears to be justified since it reduces computation time to almost one half that required for method A and for the examples presented herein gave results which are in good agreement with those of method A.

The method is outlined by merely stating the appropriate equations, the development of which is contained in appendix B. The procedure is as follows:

(1) Compute the time history of $\Delta\alpha$ by using equations (5) and (6).

(2) Determine a time history of the intermediate quantity $\Delta\xi$ from the expression

$$\Delta\xi = (1 + \lambda)\dot{\theta} - \frac{\lambda g}{V}\Delta n \tag{24}$$

or

$$\left\{\Delta\xi_i\right\} = (1 + \lambda)\left\{\dot{\theta}_i\right\} - \frac{\lambda g}{V}\left\{\Delta n_i\right\}$$

(3) Calculate time histories of $\int_0^t \Delta\alpha\, dt$, $\int_0^t \Delta\xi\, dt$, and $\int_0^t \Delta\delta\, dt$ by using the integrating matrix $\|C\|$ and the time histories of $\Delta\alpha$, $\Delta\xi$, and $\Delta\delta$.

(4) Compute $C_{m_\alpha}$, $C_{m_{\dot\theta}}$, and $C_{m_\delta}$ by least squares from the relation

$$C_{m_\alpha}\int_0^t \Delta\alpha\ dt + C_{m_{\dot\theta}}\int_0^t \Delta\xi\ dt + C_{m_\delta}\int_0^t \Delta\delta\ dt = \frac{I}{qS\bar{c}}\,\dot\theta \qquad (25)$$

or

$$\|E\|\begin{Bmatrix} C_{m_\alpha} \\ C_{m_{\dot\theta}} \\ C_{m_\delta} \end{Bmatrix} = \frac{I}{qS\bar{c}}\left\{\dot\theta_i\right\}$$

(5) Determine $C_{m_{\dot\alpha}}$ from $C_{m_{\dot\theta}}$ by using equation (23) and then determine $C_{L_{\dot\theta}}$ and $C_{L_{\dot\alpha}}$ by using equations (16) and (17).

(6) Calculate the time history of the intermediate quantity $\Delta\psi$ by inserting these values of $C_{L_{\dot\alpha}}$ and $C_{L_{\dot\theta}}$ into equation (18).

(7) Compute the values of $C_{L_\alpha}$ and $C_{L_\delta}$ from equation (19).

(8) The derivatives $\left(\frac{\partial C_L}{\partial\alpha}\right)_t$ and $\frac{\partial C_{L_t}}{\partial\delta}$ are then determined from equations (20) and (22) and the previously derived quantities.

## Method C

Method C is an extension of the method presented in reference 4. Appendix C contains the development of the pertinent equations upon which method C is based. Two basic measurements are used in this analysis, one of which must be incremental elevator deflection and the other one may be chosen from incremental load factor, pitching velocity, angle of attack, and so forth. In this paper incremental pitching velocity is used.

Two supplemental assumptions are made. The first is the relation between $C_{m_{\dot\alpha}}$ and $C_{m_{\dot\theta}}$ given in equation (23) and the second is

$$C_{m_\delta} = \frac{x_t}{\bar{c}}\,C_{L_\delta} \qquad (26)$$

The procedure for method C is as follows:

(1) Compute the stability coefficients $K_1$, $K_2$, $K_5$, and $K_6$ as outlined in step (4) of method A.

(2) Compute $C_{L_\alpha}$ from the relation

$$C_{L_\alpha} = -\frac{C_1}{2} - \sqrt{\frac{C_1{}^2}{4} - C_2} \tag{27}$$

where

$$C_1 = \frac{mV}{qS}\left[\lambda \frac{K_6}{K_5} + (1 + \lambda) \frac{Vx_t m}{I} - K_1\right] \tag{28}$$

and

$$C_2 = (1 + \lambda)\left(\frac{mV}{qS}\right)^2\left(K_2 - \frac{\lambda}{1 + \lambda} \frac{K_6}{K_5} K_1 - \frac{K_6}{K_5} \frac{x_t Vm}{I}\right) \tag{29}$$

(3) Determine $C_{m_\alpha}$ by using the expression

$$C_{m_\alpha} = -\frac{I}{qS\bar{c}} K_2 - \frac{qS}{mV} \frac{I}{\bar{c}Vm(1 + \lambda)} C_{L_\alpha}{}^2 + \frac{K_1 I}{\bar{c}Vm(1 + \lambda)} C_{L_\alpha} \tag{30}$$

(4) Calculate $C_{m_{\dot\theta}}$ from

$$C_{m_{\dot\theta}} = \frac{I}{\bar{c}Vm(1 + \lambda)}\left(C_{L_\alpha} - \frac{mV}{qS} K_1\right) \tag{31}$$

(5) Compute $C_{m_\delta}$ from

$$C_{m_\delta} = \frac{\frac{I}{qS\bar{c}} \frac{mV}{qS\bar{c}} K_6}{\frac{C_{L_\alpha}}{\bar{c}} - \frac{C_{m_\alpha}}{x_t}} \tag{32}$$

(6) Determine $C_{m_{\dot{\alpha}}}$ and $C_{L_{\delta}}$ from equations (23) and (26) by using the values of $C_{m_{\dot{\theta}}}$ and $C_{m_{\delta}}$ found in steps (4) and (5), respectively. Approximate equations for the stability derivatives are given in appendix C.

In appendix D, method C is modified slightly so that many of the stability derivatives can be obtained from a single time history. This time history must be the response to an input elevator motion of the impulse type. This modified method C comes closest to the ultimate aim of this type of analysis, namely to determine the derivatives from a single time history.

One of the important factors in obtaining reliable results with the methods outlined herein is the choice of a sufficiently small time interval $\Delta t$. In the computations using method C in this paper, in one case a time interval of $\Delta t = 0.1$ second was found to be too large to give reliable results, and a time interval of $\Delta t = 0.05$ second had to be used.

## EXAMPLES

In order to illustrate the methods outlined in the previous section as well as to compare the results obtained, a number of examples are given in which the data used are from test runs of a medium jet bomber at about the same Mach number. Methods A and C are applied to flights 1 and 2; whereas all three flights are analyzed by method B. Computations are shown in the tables for flight 1, but for the other flights only the results are given.

Table I contains the integrating matrix $\|C\|$ based on Simpson's law (ref. 3) which is used in all three methods.

The airplane characteristics and flight conditions are shown in table II(a) for all three flights. Although the geometric parameters are the same, the parameters such as weight, speed, Mach number, center-of-gravity position, and altitude vary slightly between the three runs.

In table II(b) the coefficients of the transfer function which relates pitching velocity to elevator deflection defined by equation (8) and computed by the method outlined in reference 4 are shown. These preliminary constants are required in methods A and C and the actual computations are shown in a subsequent table.

Time histories of measured and derived quantities for flight 1 are shown in table III. The quantities in columns 2, 3, 4, and 5 are measured and the other five quantities are derived from the measured quantities. In these tables more decimal places are carried in the

measured quantities than are warranted by instrument accuracy in order to assure no loss in accuracy in rounding off. The measurements of incremental tail load $\Delta L_t$ were available only for the times listed, and since these covered approximately the natural period of the short-period oscillations of the aircraft, the data were considered sufficient. More of the time histories of the other variables were available and were used.

Method A.- The principal computations illustrating method A are presented in table IV; some of the intermediate steps outlined in method A are simple computations and are therefore not included in this table. Table IV(a) is obtained by applying equation (7) to the data given in table III and illustrates step (3) of method A.

In table IV(b), the computations illustrating the determination of $C_{m_{\dot{\theta}}}$ and $C_{m_{\dot{\alpha}}}$ for steps (7) and (8) of method A are shown. Two of the columns are taken from table III and the equations upon which the computations are based are (12) and (13).

Table IV(c) illustrates the computation of $C_{m_\alpha}$ and $C_{m_\delta}$ for step (10) of method A. Two of the columns are obtained by operating on columns 6 and 2 of table III with the integrating matrix $\|C\|$ given in table I, and the other column is taken directly from table III. The computation is based on equation (15).

The refined values of $C_{L_\alpha}$ and $C_{L_\delta}$ are determined in table IV(d) by use of equation (19). Two of the columns are taken directly from table III and the other column is derived by use of equation (18).

Final results obtained with method A for the data of flights 1 and 2 after three iterations are shown in table IV(e).

Method B.- The principal computations illustrating method B are presented in table V. Again, some of the intermediate steps outlined in method B are simple computations and are therefore omitted. In table V(a) the computation demonstrating the determination of $C_{m_\alpha}$, $C_{m_{\dot{\theta}}}$, and $C_{m_\delta}$ by step (4) by using the relation (25) is shown. Three of the columns are obtained by operating on columns 6, 7, and 2 of table III with the integrating matrix $\|C\|$ given in table I.

Table V(b) illustrates step (7), the determination of $C_{L_\alpha}$ and $C_{L_\delta}$ using equation (19). Two of the columns are obtained directly from table III and the other column is derived by using equation (18).

In table V(c) final results obtained with method B for three sets of flight data are shown.

Method C.- The principal computations of method C are presented in table VI. Table VI(a) shows the computation of $K_1$, $K_2$, $K_5$, and $K_6$ from flight 1 data by the method of reference 4. The integrals in table VI(a) were computed by reading the film at 0.05-second intervals and using the integrating matrix for $\Delta t = 0.05$ second; this interval was necessary in order to obtain reasonable results for the method. Use of the time interval $\Delta t = 0.1$ second did not produce sufficiently accurate values of $K_5$ and $K_6$ in this case. Table VI(b) shows the computation of the $K$ values for flight 2 data. In this case a time interval of $\Delta t = 0.1$ second was sufficiently small to produce reliable results for method C.

In table VI(c) the final results obtained with method C for flight 1 and flight 2 data are given along with the results obtained by using the approximate formulas of appendix C.

## DISCUSSION

The three methods presented in this paper are based on the assumptions that the aircraft has two degrees of freedom (vertical motion and pitch), that the motion of the aircraft can be adequately described by the linear differential equations of motion with constant coefficients based on small-perturbation theory, that the aircraft is a rigid body with no flexibility, and that the major contribution to the damping comes from the horizontal tail. The airplane, its flight condition, and the maneuver to be analyzed must therefore fall within the realm of these assumptions; that is, the airplane should be operating under conditions in the linear range of the coefficients, the maneuver should be of the pull-up or push-down variety where little loss in airspeed occurs during the maneuver and where displacement angles are small, and the maneuver should start from a level-flight trim condition and should be in the Mach number range in which these assumptions are valid.

Since the choice of the method to be used depends primarily on the number of measurements which are available, method A is recommended when four basic measurements are available, method B when three measurements are available, and so forth. If, however, an accurate value of $\lambda$ is known in advance, then method B is recommended since it will give the most information for the least amount of work. Method C requires more work than method B, and the modified method C is not expected to be so reliable as the other methods.

In these methods sufficient data to cover the natural period of the short-period oscillations of the aircraft should be used. For highly damped motions sufficient data should be used to approach the steady-state value.

The accuracy of the results obtained from these methods is influenced considerably by errors in the instruments and in the record reading. Instruments used should be accurate, calibrated both statically and dynamically, and free from drift and hysteresis. Before an analysis is started the data should be corrected for known instrument errors; the records should then be read as carefully as possible. Measured tail-load data should be corrected for effects of inertia. The accuracy of the analysis next depends on the time interval selected for the integrating matrix and on the amount of departure from the basic assumptions. Provided the initial data are accurate, the smaller the time interval the more accurate the results. The differences between the values shown for different flights in tables IV(e), V(c), and VI(c) are believed to represent the scatter caused by effects of flexibility, minor nonlinearities, instrument errors, record-reading errors, changes in airspeed during the maneuvers, and other items which essentially depart from the basic assumptions.

As may be seen from a comparison of tables IV(a) and IV(d), the inclusion of the $C_{L_{\dot{\theta}}}$ and $C_{L_{\dot{\alpha}}}$ terms in the force equation for method A has little or no effect on $C_{L_{\alpha}}$ but has a considerable effect on $C_{L_{\delta}}$. If the $C_{L_{\dot{\alpha}}}$ and $C_{L_{\dot{\theta}}}$ terms are retained in the force equation in the development of equation (8), the form of the equation remains the same but the K values now include $C_{L_{\dot{\alpha}}}$ and $C_{L_{\dot{\theta}}}$ terms. These terms were found to have a negligible effect on the K values and their inclusion made the equations too unwieldy to handle. For the sake of completeness, the K values including the $C_{L_{\dot{\theta}}}$ and $C_{L_{\dot{\alpha}}}$ terms are given in appendix E as $\overline{K}_1$, $\overline{K}_2$, $\overline{K}_3$, $\overline{K}_4$, $\overline{K}_5$, and $\overline{K}_6$.

In the actual computation it is recommended that the simultaneous equations formed by the least-squares procedure be solved directly by the elimination of the variables or by Crout's method. (See ref. 6.) The use of a least-squares method permits the calculation of a probable error, which is an indication of the fit of the data. The expression used in computing the probable error is

$$\text{P.E.} = 0.6745\sqrt{\frac{\sum E^2}{N - \kappa}}\sqrt{B_{ii}}$$

where $B_{ii}$ is the main diagonal term of the inverted matrix of the coefficients, $E$ is the difference between the computed and measured value of the variable, $N$ is the number of cases considered in the least-squares procedure, and $\kappa$ is the number of unknowns determined by the least-squares procedure. A probable-error analysis was made of all the results using flight 1 data and these results are given in tables II(b), IV(e), V(c), and VI(c). This probable-error analysis indicates that all the derivatives determined by method A with the exception of $C_{L_{\delta}}$ appear to be more accurate than the derivatives determined by method B or C; it also indicates that the derivatives determined by methods B and C appear to be of the same order of accuracy.

When the computed stability derivatives are substituted back into the equations of motion, the method that uses the most measurements and has the fewest restraints imposed on it would be expected to produce the most accurate results and give the best fit to the original data. This might not be the case, as illustrated in figure 2 which compares the fit of the measured data with the computed data for the three methods presented. The results for method A are more accurate for the data herein than the results for method C, but the fit of the incremental-pitching-velocity curve for method C is as good as if not better than the fit for method A or method B. It appears in general that the more coefficients determined from a single time history the better will be the fit of the data but the less accurate will be the coefficients determined. The fit of the data is interesting since the three methods presented are essentially curve-fitting processes in which the longitudinal equations of motion are used to fit the flight data. A good fit indicates that the equations of motion and assumptions used adequately fit the data and the coefficients determined, if inserted in the equations of motion, will reproduce the motions of the aircraft.

In figure 2 the incremental tail load shown for method B was computed by using the stability derivatives determined from the time histories of incremental load factor and pitching velocity. In method C the incremental load factor and tail load presented were computed by using the derivatives determined from the pitching velocity. These time histories indicate how well the derivatives determined on the basis of the measurements recorded by one set of instruments will predict the measurements recorded by a different set of instruments. In the case of method C the agreement is good; in the case of method B it appears that a more realistic value of $\lambda$ than 0.5 should be used. Method B is more sensitive to $\lambda$ than method C is.

Although not presented, the derivatives determined from flight 2 by methods A and C were used to predict the motions of the aircraft for flight 1. A comparison was then made with the actual flight 1 motions and it was found that the predicted motions and the actual motions were

in good agreement. These results verify the validity of the method outlined herein as applied to the example airplane.

A possibility for a further generalized method which would include damping effects of wing and fuselage and therefore would make the method applicable to the case of swept-wing airplanes may be realized by combining features of method C with method A in the following manner. Equation (A20) may be written in the form

$$C_{m_{\dot{\theta}_t}}\left\{\dot{\theta} - \frac{V}{x_t}\eta_t\,\Delta\alpha\right\} + C_{m_{\dot{\alpha}}}\left\{\frac{V}{x_t}\Delta\alpha + \dot{\alpha}\right\} = \left\{\frac{x_t}{qS\bar{c}}\Delta L_t - \frac{x_t}{\bar{c}}C_{L_\delta}\,\Delta\delta\right\}$$

Now $C_{m_{\dot{\theta}_t}}$ and $C_{m_{\dot{\alpha}}}$ may be evaluated by a least-squares procedure, provided an accurate value of $C_{L_\delta}$ is available or can be determined. Examination of the probable errors for $C_{L_\delta}$ given in tables IV(e) and VI(c) indicate that, in the case of the medium jet bomber used in the calculations herein, the more accurate value of $C_{L_\delta}$ is determined by method C by using equations (C12) and (C4). It is believed this will generally be the case for the derivative $C_{L_\delta}$. It might also be noted that this value of $C_{L_\delta}$ will provide more rapid convergence of the iterative procedure of method A. The usual assumption is made that the contribution of wing and fuselage to $C_{m_{\dot{\alpha}}}$ is negligible. Then $C_{m_{\dot{\theta}}}$ can be computed through the use of the value of $C_{m_{\dot{\alpha}}}$ computed by the above procedure and equation (A22).

Possibilities for further investigation are to expand the method to include flexibility effects and the effects of higher-order derivatives and to extend the method to the case where the initial conditions are known but are not necessarily zero; that is, the maneuvers do not start from level-flight trim conditions. The methods could also be extended to other configurations such as canard aircraft and tailless aircraft, and perhaps a similar analysis could be made of the lateral motion of an aircraft to determine the lateral derivatives.

## CONCLUDING REMARKS

An analysis of longitudinal-stability derivatives by three separate methods has been presented and applied to flight data. Method A, the most general method, requires four measurements in time-history form and permits computation of all the longitudinal-stability derivatives; it also requires the most computing time and gives the most accurate answers. Method B, which requires three measurements in time-history

form and one supplemental assumption, namely $\frac{C_{m_{\dot{\alpha}}}}{C_{m_{\dot{\theta}}}}$ = Constant (where $C_{m_{\dot{\alpha}}}$ and $C_{m_{\dot{\theta}}}$ are the pitching-moment derivatives), gives the most information for the least amount of work and gives results which are in good agreement with those of method A. Method C requires two measurements in time-history form and two supplementary assumptions, namely $\frac{C_{m_{\dot{\alpha}}}}{C_{m_{\dot{\theta}}}}$ = Constant and $C_{m_{\delta}} = \frac{x_t}{\bar{c}} C_{L_{\delta}}$ (where $C_{m_{\delta}}$ and $C_{L_{\delta}}$ are the elevator-effectiveness derivatives, $x_t$ is the tail length, and $\bar{c}$ is the mean aerodynamic chord).

The results obtained for the methods presented depend in a large measure on accurate instrument measurements and require considerable computation to yield adequate engineering answers. Since, however, the present trend is toward increased instrument accuracy and expanded facilities for machine computation, this direction appears to be the one in which flight-data analysis should proceed.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., August 15, 1952.

# APPENDIX A

## DETERMINATION OF LONGITUDINAL-STABILITY DERIVATIVES BY USING FOUR MEASUREMENTS IN TIME-HISTORY FORM

### Equations of Motion

The equations of motion for small vertical-plane disturbances may be stated (see fig. 1 for definition) as

$$\frac{W}{g} V\dot{\gamma} = \left(\frac{\partial C_L}{\partial \alpha}\right)_{WB} qS\ \Delta\alpha + \left(\frac{\partial C_L}{\partial \alpha}\right)_t \eta_t q S_t\ \Delta\alpha_t + \frac{\partial C_{L_t}}{\partial \delta} \eta_t q S_t\ \Delta\delta \qquad \text{(A1)}$$

$$I\ddot{\theta} = \left(\frac{\partial C_m}{\partial \alpha}\right)_{WB} qS\bar{c}\ \Delta\alpha + \left(\frac{\partial C_L}{\partial \alpha}\right)_t \eta_t q S_t x_t\ \Delta\alpha_t + \frac{\partial C_{m_t}}{\partial \delta} \eta_t q S_t \bar{c}_t\ \Delta\delta + \frac{\partial C_{L_t}}{\partial \delta} \eta_t q S_t x_t\ \Delta\delta \qquad \text{(A2)}$$

where

$$\Delta\alpha_t = \Delta\alpha\left(1 - \frac{\partial \epsilon}{\partial \alpha}\right) - \dot{\alpha}\frac{x_t}{V}\frac{\partial \epsilon}{\partial \alpha} - \dot{\theta}\frac{x_t}{V}\frac{1}{\sqrt{\eta_t}} \qquad \text{(A3)}$$

These equations are for a rigid body and are based on the usual assumptions of linearity, small angles, and no loss in airspeed during the maneuver. It should be noted that the variables are all in incremental form measured from a steady-flight trim condition.

Substituting equation (A3) into equation (A1) results in

$$\frac{W}{g} V\dot{\gamma} = \Delta\alpha\left[\left(\frac{\partial C_L}{\partial \alpha}\right)_{WB} qS + \left(\frac{\partial C_L}{\partial \alpha}\right)_t \eta_t q S_t \left(1 - \frac{\partial \epsilon}{\partial \alpha}\right)\right] - \dot{\alpha}\left(\frac{\partial C_L}{\partial \alpha}\right)_t \eta_t q S_t \frac{x_t}{V}\frac{\partial \epsilon}{\partial \alpha} -$$

$$\dot{\theta}\left(\frac{\partial C_L}{\partial \alpha}\right)_t \eta_t q S_t \frac{x_t}{V}\frac{1}{\sqrt{\eta_t}} + \frac{\partial C_{L_t}}{\partial \delta} \eta_t q S_t\ \Delta\delta \qquad \text{(A4)}$$

Since

$$\Delta n = \frac{V}{g} \dot{\gamma} \tag{A5}$$

equation (A4) can be expressed as

$$\frac{W}{qS} \Delta n = C_{L_\alpha} \Delta\alpha + C_{L_{\dot{\alpha}}}\dot{\alpha} + C_{L_{\dot{\theta}}}\dot{\theta} + C_{L_\delta} \Delta\delta \tag{A6}$$

where

$$C_{L_\alpha} = \left(\frac{\partial C_L}{\partial \alpha}\right)_{WB} + \left(\frac{\partial C_L}{\partial \alpha}\right)_t \eta_t \frac{S_t}{S}\left(1 - \frac{\partial \epsilon}{\partial \alpha}\right) \tag{A7a}$$

$$C_{L_{\dot{\alpha}}} = -\left(\frac{\partial C_L}{\partial \alpha}\right)_t \eta_t \frac{S_t}{S} \frac{x_t}{V} \frac{\partial \epsilon}{\partial \alpha} \tag{A7b}$$

$$C_{L_{\dot{\theta}}} = -\left(\frac{\partial C_L}{\partial \alpha}\right)_t \eta_t \frac{S_t}{S} \frac{x_t}{V} \frac{1}{\sqrt{\eta_t}} \tag{A7c}$$

$$C_{L_\delta} = \frac{\partial C_{L_t}}{\partial \delta} \eta_t \frac{S_t}{S} \tag{A7d}$$

Substituting equation (A3) into equation (A2) gives

$$\frac{I}{qS\bar{c}} \ddot{\theta} = \left(\frac{\partial C_m}{\partial \alpha}\right)_{WB} \Delta\alpha + \left(\frac{\partial C_L}{\partial \alpha}\right)_t \eta_t \frac{S_t}{S} \frac{x_t}{\bar{c}}\left[\Delta\alpha\left(1 - \frac{\partial \epsilon}{\partial \alpha}\right) - \dot{\alpha} \frac{x_t}{V} \frac{\partial \epsilon}{\partial \alpha} - \dot{\theta} \frac{x_t}{V} \frac{1}{\sqrt{\eta_t}}\right] +$$

$$\frac{\partial C_{m_t}}{\partial \delta} \eta_t \frac{S_t}{S} \frac{\bar{c}_t}{\bar{c}} \Delta\delta + \frac{\partial C_{L_t}}{\partial \delta} \eta_t \frac{S_t}{S} \frac{x_t}{\bar{c}} \Delta\delta \tag{A8}$$

or

$$\frac{I\ddot{\theta}}{qS\bar{c}} = \Delta\alpha\left[\left(\frac{\partial C_m}{\partial\alpha}\right)_{WB} + \left(\frac{\partial C_L}{\partial\alpha}\right)_t \eta_t \frac{S_t}{S}\frac{x_t}{\bar{c}}\left(1 - \frac{\partial\epsilon}{\partial\alpha}\right)\right] + \dot{\alpha}\left[-\left(\frac{\partial C_L}{\partial\alpha}\right)_t \eta_t \frac{S_t}{S}\frac{{x_t}^2}{\bar{c}V}\frac{\partial\epsilon}{\partial\alpha}\right] +$$

$$\dot{\theta}\left[-\left(\frac{\partial C_L}{\partial\alpha}\right)_t \eta_t \frac{S_t}{S}\frac{{x_t}^2}{\bar{c}V}\frac{1}{\sqrt{\eta_t}}\right] + \Delta\delta\left(\frac{\partial C_{L_t}}{\partial\delta}\eta_t\frac{S_t}{S}\frac{x_t}{\bar{c}} + \frac{\partial C_{m_t}}{\partial\delta}\eta_t\frac{S_t}{S}\frac{\bar{c}_t}{\bar{c}}\right) \quad \text{(A9)}$$

which can be expressed as

$$\frac{I\ddot{\theta}}{qS\bar{c}} = C_{m_\alpha}\Delta\alpha + C_{m_{\dot{\alpha}}}\dot{\alpha} + C_{m_{\dot{\theta}}}\dot{\theta} + C_{m_\delta}\Delta\delta \quad \text{(A10)}$$

where

$$C_{m_\alpha} = \left(\frac{\partial C_m}{\partial\alpha}\right)_{WB} + \left(\frac{\partial C_L}{\partial\alpha}\right)_t \eta_t \frac{S_t}{S}\frac{x_t}{\bar{c}}\left(1 - \frac{\partial\epsilon}{\partial\alpha}\right) \quad \text{(A11a)}$$

$$C_{m_{\dot{\alpha}}} = -\left(\frac{\partial C_L}{\partial\alpha}\right)_t \eta_t \frac{S_t}{S}\frac{{x_t}^2}{\bar{c}V}\frac{\partial\epsilon}{\partial\alpha} \quad \text{(A11b)}$$

$$C_{m_{\dot{\theta}}} = -\left(\frac{\partial C_L}{\partial\alpha}\right)_t \eta_t \frac{S_t}{S}\frac{{x_t}^2}{\bar{c}V}\frac{1}{\sqrt{\eta_t}} \quad \text{(A11c)}$$

$$C_{m_\delta} = \frac{\partial C_{L_t}}{\partial\delta}\eta_t\frac{S_t}{S}\frac{x_t}{\bar{c}} + \frac{\partial C_{m_t}}{\partial\delta}\eta_t\frac{S_t}{S}\frac{\bar{c}_t}{\bar{c}} \quad \text{(A11d)}$$

From an examination of equations (A11) and (A7), the following relations are seen to exist:

$$C_{m_{\dot{\alpha}}} = \frac{x_t}{\bar{c}} C_{L_{\dot{\alpha}}} \quad \text{(A12a)}$$

$$C_{m_{\dot{\theta}}} = \frac{x_t}{\bar{c}} C_{L_{\dot{\theta}}} \tag{A12b}$$

$$C_{m_{\dot{\alpha}}} = \sqrt{\eta_t} \frac{\partial\epsilon}{\partial\alpha} C_{m_{\dot{\theta}}} \tag{A12c}$$

Equations (A6) and (A10) are linearly dependent in the present form and must be suitably altered to be put into a computational form. As is well-known, $C_{L_{\dot{\theta}}}$ and $C_{L_{\dot{\alpha}}}$ are small, and initially for computational purposes the force equation can be expressed as

$$\frac{W}{qS} \Delta n = C_{L_\alpha} \Delta\alpha + C_{L_\delta} \Delta\delta \tag{A13}$$

The derivatives $C_{L_{\dot{\theta}}}$ and $C_{L_{\dot{\alpha}}}$ can be determined by means of equations (A12a) and (A12b) after $C_{m_{\dot{\theta}}}$ and $C_{m_{\dot{\alpha}}}$ are determined.

Equations (A13) and (A10) are now in the identical form of the equations of motion developed in reference 7. From figure 1 the following relation is seen to exist:

$$\Delta\theta = \Delta\alpha + \Delta\gamma \tag{A14}$$

As demonstrated in reference 4, equations (A10), (A13), and (A14) may be solved simultaneously to obtain the relation

$$\ddot{\theta} + K_1\dot{\theta} + K_2 \Delta\theta = K_5 \Delta\delta + K_6 \int_0^t \Delta\delta \, dt \tag{A15}$$

which may be expressed in integral form as

$$\dot{\theta} + K_1 \Delta\theta + K_2 \int_0^t \Delta\theta \, dt = K_5 \int_0^t \Delta\delta \, dt + K_6 \int_0^t \int_0^\tau \Delta\delta \, d\tau \, dt$$

where

$$K_1 = \frac{qS}{V}\left[\frac{C_{L_\alpha}}{m} - \frac{\bar{c}V}{I}\left(C_{m_{\dot{\alpha}}} + C_{m_{\dot{\theta}}}\right)\right] \tag{A16a}$$

$$K_2 = -\frac{qS\bar{c}}{I}\left(C_{m_\alpha} + C_{m_{\dot{\theta}}}\frac{C_{L_\alpha}qS}{mV}\right) \tag{A16b}$$

$$K_5 = \frac{qS\bar{c}}{I}\left(C_{m_\delta} - \frac{qS}{mV}C_{L_\delta}C_{m_{\dot{\alpha}}}\right) \tag{A16c}$$

$$K_6 = \frac{qS\bar{c}}{I}\frac{qS}{mV}\left(C_{L_\alpha}C_{m_\delta} - C_{L_\delta}C_{m_\alpha}\right) \tag{A16d}$$

By using the matrix method of reference 4, $K_1$, $K_2$, $K_5$, and $K_6$ may be evaluated from the time-history measurements of pitching velocity and elevator angle.

## Method of Separating $C_{m_{\dot{\theta}}}$ and $C_{m_{\dot{\alpha}}}$

This method of separating $C_{m_{\dot{\theta}}}$ and $C_{m_{\dot{\alpha}}}$ applies only to conventional aircraft configurations equipped with a horizontal tail surface located to the rear of the wing so that the major contribution to damping in pitch is due to the horizontal tail.

In order to separate $C_{m_{\dot{\theta}}}$ and $C_{m_{\dot{\alpha}}}$, the tail-load equation is developed into a form suitable for computing $C_{m_{\dot{\theta}}}$ separately in the following manner:

The incremental tail load is given by

$$\Delta L_t = \left(\frac{\partial C_L}{\partial \alpha}\right)_t \eta_t q S_t \, \Delta\alpha_t + \frac{\partial C_{L_t}}{\partial \delta}\eta_t S_t q \, \Delta\delta \tag{A17}$$

Substituting equation (A3) into equation (A17) gives

$$\Delta L_t = \left(\frac{\partial C_L}{\partial \alpha}\right)_t \eta_t q S_t\left[\Delta\alpha\left(1 - \frac{\partial \epsilon}{\partial \alpha}\right) - \dot{\alpha}\frac{x_t}{V}\frac{\partial \epsilon}{\partial \alpha} - \dot{\theta}\frac{x_t}{V}\frac{1}{\sqrt{\eta_t}}\right] + \frac{\partial C_{L_t}}{\partial \delta}\eta_t S_t q \, \Delta\delta \tag{A18}$$

or

$$\frac{\Delta L_t}{qS} = \left(\frac{\partial C_L}{\partial \alpha}\right)_t \eta_t \frac{S_t}{S}\left(1 - \frac{\partial \epsilon}{\partial \alpha}\right)\Delta\alpha - \left(\frac{\partial C_L}{\partial \alpha}\right)_t \eta_t \frac{S_t}{S} \frac{x_t}{V} \frac{\partial \epsilon}{\partial \alpha} \dot{\alpha} -$$

$$\left(\frac{\partial C_L}{\partial \alpha}\right)_t \eta_t \frac{S_t}{S} \frac{x_t}{V} \frac{1}{\sqrt{\eta_t}} \dot{\theta} + \frac{\partial C_{L_t}}{\partial \delta} \eta_t \frac{S_t}{S} \Delta\delta \tag{A19}$$

which can be expressed as

$$\frac{\Delta L_t}{qS} = \left[-\frac{\bar{c}V}{x_t^2}\left(\sqrt{\eta_t}\, C_{m_{\dot{\theta}}} - C_{m_{\dot{\alpha}}}\right)\right]\Delta\alpha + \frac{\bar{c}}{x_t}\left(C_{m_{\dot{\alpha}}}\dot{\alpha} + C_{m_{\dot{\theta}}}\dot{\theta}\right) + C_{L_\delta}\,\Delta\delta \tag{A20}$$

From equation (A14) it can be seen that

$$C_{m_{\dot{\alpha}}}\dot{\alpha} + C_{m_{\dot{\theta}}}\dot{\theta} = \left(C_{m_{\dot{\alpha}}} + C_{m_{\dot{\theta}}}\right)\dot{\alpha} + C_{m_{\dot{\theta}}}\dot{\gamma} \tag{A21}$$

but from equation (A16a)

$$C_{m_{\dot{\alpha}}} + C_{m_{\dot{\theta}}} = \frac{I}{\bar{c}Vm} C_{L_\alpha} - \frac{I}{qS\bar{c}} K_1 = K_{10} \tag{A22}$$

Therefore,

$$C_{m_{\dot{\alpha}}}\dot{\alpha} + C_{m_{\dot{\theta}}}\dot{\theta} = K_{10}\dot{\alpha} + C_{m_{\dot{\theta}}}\dot{\gamma} \tag{A23}$$

Substituting equations (A22) and (A23) into equation (A20) gives

$$\frac{\Delta L_t}{qS} = \left[-\frac{\bar{c}V}{x_t^2}\left(\sqrt{\eta_t}\, C_{m_{\dot{\theta}}} - K_{10} + C_{m_{\dot{\theta}}}\right)\right]\Delta\alpha + \frac{\bar{c}}{x_t}\left(K_{10}\dot{\alpha} + C_{m_{\dot{\theta}}}\dot{\gamma}\right) + C_{L_\delta}\,\Delta\delta \tag{A24}$$

which can be expressed as

$$\frac{\Delta L_t}{qS} - \frac{\bar{c}V}{x_t^2} K_{10} \Delta\alpha - \frac{\bar{c}}{x_t} K_{10}\dot{\alpha} - C_{L_\delta} \Delta\delta = C_{m_{\dot{\theta}}}\left[\frac{\bar{c}}{x_t} \dot{\gamma} - \frac{\bar{c}V}{x_t^2}\left(\sqrt{\eta_t} + 1\right)\Delta\alpha\right] \tag{A25}$$

or

$$\Delta\mu = C_{m_{\dot{\theta}}} \Delta\varphi \tag{A26}$$

where

$$\Delta\mu = \frac{\Delta L_t}{qS} - \frac{\bar{c}V}{x_t^2} K_{10} \Delta\alpha - \frac{\bar{c}}{x_t} K_{10}\dot{\alpha} - C_{L_\delta} \Delta\delta \tag{A27}$$

and

$$\Delta\varphi = \frac{\bar{c}}{x_t} \dot{\gamma} - \frac{\bar{c}V}{x_t^2}\left(\sqrt{\eta_t} + 1\right)\Delta\alpha = \frac{\bar{c}}{x_t}\left[\frac{g}{V} \Delta n - \frac{V}{x_t}\left(\sqrt{\eta_t} + 1\right)\Delta\alpha\right] \tag{A28}$$

From equations (A14) and (A5) the following relations are self-evident:

$$\dot{\alpha} = \dot{\theta} - \frac{g}{V} \Delta n$$

$$\Delta\alpha = \Delta\theta - \frac{g}{V}\int_0^t \Delta n \, dt \tag{A29}$$

The relations needed to determine the longitudinal-stability derivatives from the flight measurements have now been developed from the equations of motion; it remains to express the pertinent relations in matrix notation.

## Matrix Form of the Equations

A powerful tool for data analysis is provided by matrix methods since tabulated time histories may be conveniently carried in the equations of motion. In the matrix solutions using data, it is well-known that numerical differentiation is inherently more inaccurate than the corresponding integration process. For this reason, whenever necessary, the differential equations are expressed in integral form. The first

step in matrix solutions is to tabulate the recorded values of the basic variables at a number of points $t_0$, $t_1$, $t_2$, $t_3$ . . . along a given time history as in table III, the interval of time used in most of the computations in the paper being $\Delta t = 0.1$ second. These tabulations then become the various column matrices $\Delta\delta_i$, $\Delta L_{t_i}$, $\Delta n_i$, and $\dot{\theta}_i$. In certain cases smaller time intervals must be used to get reliable results. Another means of getting more accuracy is to use integrating matrices based on cubic or quartic curves faired through the data in place of the parabolic curves.

The four basic measurements used in the development herein are incremental load factor, pitching velocity, tail load, and elevator angle. By use of equation (A29), the time history of incremental angle of attack is computed. Equation (A13) may be expressed as

$$\begin{Vmatrix} \Delta\alpha_0 & \Delta\delta_0 \\ \Delta\alpha_1 & \Delta\delta_1 \\ \Delta\alpha_2 & \Delta\delta_2 \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \Delta\alpha_n & \Delta\delta_n \end{Vmatrix} \begin{Bmatrix} C_{L_\alpha} \\ C_{L_\delta} \end{Bmatrix} = \frac{W}{qS} \begin{Bmatrix} \Delta n_0 \\ \Delta n_1 \\ \Delta n_2 \\ \cdot \\ \cdot \\ \cdot \\ \Delta n_n \end{Bmatrix} \tag{A30}$$

or

$$\lVert B \rVert \begin{Bmatrix} C_{L_\alpha} \\ C_{L_\delta} \end{Bmatrix} = \frac{W}{qS} \left\{ \Delta n_i \right\} \tag{A31}$$

Applying least squares, which in matrix notation involves premultiplication of matrix B by its transpose B', to equation (A31) yields

$$\left[ B'B \right] \begin{Bmatrix} C_{L_\alpha} \\ C_{L_\delta} \end{Bmatrix} = \frac{W}{qS} \left\{ B'\Delta n_i \right\} \tag{A32}$$

for which the solution is

$$\begin{Bmatrix} C_{L_\alpha} \\ C_{L_\delta} \end{Bmatrix} = \left[B'B\right]^{-1} \frac{W}{qS} \left\{B'\Delta n_i\right\} \tag{A33}$$

By the method of reference 4, compute $K_1$, $K_2$, $K_5$, and $K_6$ from the time histories of the pitching velocity and elevator angle. The value of $K_{10}$ can be obtained from equation (A22). Time histories of the derived $\Delta\mu$ and $\Delta\varphi$ functions can now be computed by using equations (A27) and (A28) since the value of $C_{L_\delta}$ has been computed from equation (A33).

Equation (A26) becomes

$$\left\{\Delta\mu_i\right\} = C_{m_{\dot{\theta}}} \left\{\Delta\varphi_i\right\} \tag{A34}$$

Applying least squares to equation (A34) results in

$$C_{m_{\dot{\theta}}} = \frac{\sum_0^t \left(\Delta\mu_i \; \Delta\varphi_i\right)}{\sum_0^t \left(\Delta\varphi_i\right)^2} \tag{A35}$$

From equation (A22) $C_{m_{\dot{\alpha}}}$ is obtained as

$$C_{m_{\dot{\alpha}}} = K_{10} - C_{m_{\dot{\theta}}} \tag{A36}$$

Equation (A10) is now expressed in integral form as

$$\frac{I}{qS\bar{c}} \dot{\theta} = C_{m_\alpha} \int_0^t \Delta\alpha \; dt + C_{m_{\dot{\alpha}}} \Delta\alpha + C_{m_{\dot{\theta}}} \Delta\theta + C_{m_\delta} \int_0^t \Delta\delta \; dt \tag{A37}$$

which can be rewritten as

$$\frac{I}{qS\bar{c}}\dot{\theta} - C_{m_{\dot{\alpha}}}\Delta\alpha - C_{m_{\dot{\theta}}}\Delta\theta = C_{m_{\alpha}}\int_{0}^{t}\Delta\alpha\,dt + C_{m_{\delta}}\int_{0}^{t}\Delta\delta\,dt \tag{A38}$$

Now if

$$\Delta\sigma = \frac{I}{qS\bar{c}}\dot{\theta} - C_{m_{\dot{\alpha}}}\Delta\alpha - C_{m_{\dot{\theta}}}\Delta\theta \tag{A39}$$

then a time history of $\Delta\sigma$ can be obtained and equation (A38) can be put in the form

$$\left\|\begin{matrix} \int_0^{t_0}\Delta\alpha\,dt & \int_0^{t_0}\Delta\delta\,dt \\ \int_0^{t_1}\Delta\alpha\,dt & \int_0^{t_1}\Delta\delta\,dt \\ \int_0^{t_2}\Delta\alpha\,dt & \int_0^{t_2}\Delta\delta\,dt \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \int_0^{t_n}\Delta\alpha\,dt & \int_0^{t_n}\Delta\delta\,dt \end{matrix}\right\| \begin{Bmatrix} C_{m_{\alpha}} \\ C_{m_{\delta}} \end{Bmatrix} = \begin{Bmatrix} \Delta\sigma_0 \\ \Delta\sigma_1 \\ \Delta\sigma_2 \\ \cdot \\ \cdot \\ \cdot \\ \Delta\sigma_n \end{Bmatrix} \tag{A40}$$

or

$$\|D\| \begin{Bmatrix} C_{m_{\alpha}} \\ C_{m_{\delta}} \end{Bmatrix} = \left\{\Delta\sigma_i\right\} \tag{A41}$$

Applying least squares to equation (A41) gives

$$\left[D'D\right] \begin{Bmatrix} C_{m_{\alpha}} \\ C_{m_{\delta}} \end{Bmatrix} = \left\{D'\Delta\sigma_i\right\} \tag{A42}$$

and the solution is

$$\begin{Bmatrix} C_{m_\alpha} \\ C_{m_\delta} \end{Bmatrix} = \left[D'D\right]^{-1} \left\{D'\Delta\sigma_i\right\} \tag{A43}$$

In order to include the effects of the $C_{L_{\dot{\theta}}}$ and $C_{L_{\dot{\alpha}}}$ terms initially omitted in the force equation, equation (A6) is rewritten as

$$\Delta\psi = C_{L_\alpha} \Delta\alpha + C_{L_\delta} \Delta\delta \tag{A44}$$

where

$$\Delta\psi = \frac{W}{qS} \Delta n - C_{L_{\dot{\theta}}}\dot{\theta} - C_{L_{\dot{\alpha}}}\dot{\alpha} \tag{A45}$$

Method A may now be iterated to obtain more refined values of the derivatives. The values of $C_{L_{\dot{\theta}}}$ and $C_{L_{\dot{\alpha}}}$ determined by equations (A35), (A36), (A12a), and (A12b) are inserted into equation (A45), and a time history of $\Delta\psi$ is computed. New values of $C_{L_\alpha}$ and $C_{L_\delta}$ are computed from

$$\left[B'B\right] \begin{Bmatrix} C_{L_\alpha} \\ C_{L_\delta} \end{Bmatrix} = \left\{B'\Delta\psi_i\right\} \tag{A46}$$

or

$$\begin{Bmatrix} C_{L_\alpha} \\ C_{L_\delta} \end{Bmatrix} = \left[B'B\right]^{-1} \left\{B'\Delta\psi_i\right\}$$

If these values of $C_{L_\alpha}$ and $C_{L_\delta}$ are used, a new value of $K_{10}$ and a new time history of $\Delta\mu$ can be computed, which, if inserted into equations (A34) and (A36), yield new values of $C_{m_{\dot{\theta}}}$ and $C_{m_{\dot{\alpha}}}$. The derivatives $C_{L_{\dot{\alpha}}}$ and $C_{L_{\dot{\theta}}}$ are again determined from equations (A12a) and

(A12b) and a new time history of $\Delta\psi$ is computed by using equation (A45); refined $C_{L_\alpha}$ and $C_{L_\delta}$ derivatives are found from equation (A46). The process converges rapidly. After it has converged, compute $C_{m_\alpha}$ and $C_{m_\delta}$ from equation (A43). Thus far $C_{L_\alpha}$, $C_{L_{\dot\theta}}$, $C_{L_{\dot\alpha}}$, $C_{L_\delta}$, $C_{m_\alpha}$, $C_{m_{\dot\theta}}$, $C_{m_{\dot\alpha}}$, and $C_{m_\delta}$ have been determined. Then $\left(\frac{\partial C_L}{\partial \alpha}\right)_t$ may be determined by rewriting equation (A11c) as

$$\left(\frac{\partial C_L}{\partial \alpha}\right)_t = -\frac{S\bar{c}V}{S_t x_t^2 \sqrt{\eta_t}} C_{m_{\dot\theta}}$$

An examination of equation (A12c) shows that

$$\frac{\partial \epsilon}{\partial \alpha} = \frac{1}{\sqrt{\eta_t}} \frac{C_{m_{\dot\alpha}}}{C_{m_{\dot\theta}}}$$

and from equation (A7d)

$$\frac{\partial C_{L_t}}{\partial \delta} = \frac{S}{\eta_t S_t} C_{L_\delta}$$

All the longitudinal-stability derivatives are now determined.

## APPENDIX B

## DETERMINATION OF LONGITUDINAL-STABILITY DERIVATIVES BY USING THREE MEASUREMENTS IN TIME-HISTORY FORM AND ONE SUPPLEMENTAL ASSUMPTION

The three basic measurements used in method B are incremental load factor, pitching velocity, and elevator angle. The supplemental assumption made is that $C_{m_{\dot{\alpha}}}/C_{m_{\dot{\theta}}}$ is a constant, that is,

$$C_{m_{\dot{\alpha}}} = \lambda C_{m_{\dot{\theta}}} \tag{B1}$$

For a first approximation the constant is assumed to be equal to 1/2 (see ref. 5).

If the definition

$$\Delta\xi = \lambda\dot{\alpha} + \dot{\theta} = (1 + \lambda)\dot{\theta} - \lambda \frac{g}{V}\Delta n \tag{B2}$$

is adopted, a time history of $\Delta\xi$ may be computed. Then

$$C_{m_{\dot{\theta}}}\dot{\theta} + C_{m_{\dot{\alpha}}}\dot{\alpha} = C_{m_{\dot{\theta}}} \Delta\xi \tag{B3}$$

The integral form of the moment equation (A37) can then be written

$$\frac{I}{qS\bar{c}}\dot{\theta} = C_{m_{\alpha}}\int_0^t \Delta\alpha \, dt + C_{m_{\dot{\theta}}}\int_0^t \Delta\xi \, dt + C_{m_{\delta}}\int_0^t \Delta\delta \, dt \tag{B4}$$

If time histories of incremental load factor, pitching velocity, and elevator angle are measured and $W$, $q$, $S$, $V$, $\bar{c}$, and $I$ are known, then equation (B4) can be put into matrix form and used directly to compute $C_{m_{\alpha}}$, $C_{m_{\dot{\theta}}}$, and $C_{m_{\delta}}$.

The derivative $C_{m_{\dot{\alpha}}}$ is derived from $C_{m_{\dot{\theta}}}$ by using equation (B1) and $C_{L_{\dot{\alpha}}}$ and $C_{L_{\dot{\theta}}}$ are computed from equations (A12a) and (A12b)

$$C_{L_{\dot{\alpha}}} = \frac{\bar{c}}{x_t} C_{m_{\dot{\alpha}}}$$

$$C_{L_{\dot{\theta}}} = \frac{\bar{c}}{x_t} C_{m_{\dot{\theta}}}$$

These values of $C_{L_{\dot{\alpha}}}$ and $C_{L_{\dot{\theta}}}$ are then inserted into equation (A45)

$$\Delta\psi = \frac{W}{qS} \Delta n - C_{L_{\dot{\theta}}}\dot{\theta} - C_{L_{\dot{\alpha}}}\dot{\alpha}$$

and a time history of $\Delta\psi$ is computed. The values of $C_{L_{\alpha}}$ and $C_{L_{\delta}}$ are then computed from equation (A44)

$$\Delta\psi = C_{L_{\alpha}} \Delta\alpha + C_{L_{\delta}} \Delta\delta$$

Equation (B4) may be expressed in matrix form as

$$\left\| \begin{matrix} \int_0^{t_0} \Delta\alpha \, dt & \int_0^{t_0} \Delta\xi \, dt & \int_0^{t_0} \Delta\delta \, dt \\ \int_0^{t_1} \Delta\alpha \, dt & \int_0^{t_1} \Delta\xi \, dt & \int_0^{t_1} \Delta\delta \, dt \\ \int_0^{t_2} \Delta\alpha \, dt & \int_0^{t_2} \Delta\xi \, dt & \int_0^{t_2} \Delta\delta \, dt \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \int_0^{t_n} \Delta\alpha \, dt & \int_0^{t_n} \Delta\xi \, dt & \int_0^{t_n} \Delta\delta \, dt \end{matrix} \right\| \begin{Bmatrix} C_{m_{\alpha}} \\ C_{m_{\dot{\theta}}} \\ C_{m_{\delta}} \end{Bmatrix} = \frac{I}{qS\bar{c}} \begin{Bmatrix} \dot{\theta}_0 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \\ \cdot \\ \cdot \\ \cdot \\ \dot{\theta}_n \end{Bmatrix} \qquad \text{(B5)}$$

or

$$\|E\| \begin{Bmatrix} C_{m_\alpha} \\ C_{m_{\dot\theta}} \\ C_{m_\delta} \end{Bmatrix} = \frac{I}{qS\bar{c}} \left\{\ddot\theta_i\right\} \tag{B6}$$

Applying least squares yields

$$\left[E'E\right] \begin{Bmatrix} C_{m_\alpha} \\ C_{m_{\dot\theta}} \\ C_{m_\delta} \end{Bmatrix} = \frac{I}{qS\bar{c}} \left\{E'\ddot\theta_i\right\} \tag{B7}$$

and solving for the derivatives gives

$$\begin{Bmatrix} C_{m_\alpha} \\ C_{m_{\dot\theta}} \\ C_{m_\delta} \end{Bmatrix} = \left[E'E\right]^{-1} \frac{I}{qS\bar{c}} \left\{E'\ddot\theta_i\right\} \tag{B8}$$

Equation (A44) expressed in matrix form is

$$\left\| \begin{matrix} \Delta\alpha_0 & \Delta\delta_0 \\ \Delta\alpha_1 & \Delta\delta_1 \\ \Delta\alpha_2 & \Delta\delta_2 \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \Delta\alpha_n & \Delta\delta_n \end{matrix} \right\| \begin{Bmatrix} C_{l_\alpha} \\ C_{l_\delta} \end{Bmatrix} = \begin{Bmatrix} \Delta\psi_0 \\ \Delta\psi_1 \\ \Delta\psi_2 \\ \cdot \\ \cdot \\ \cdot \\ \Delta\psi_n \end{Bmatrix} \tag{B9}$$

or

$$\|B\| \begin{Bmatrix} C_{L_\alpha} \\ C_{L_\delta} \end{Bmatrix} = \left\{\Delta\psi_i\right\} \tag{B10}$$

Applying least squares to equation (B10) gives

$$\left[B'B\right] \begin{Bmatrix} C_{L_\alpha} \\ C_{L_\delta} \end{Bmatrix} = \left\{B'\Delta\psi_i\right\} \tag{B11}$$

and solving for the unknowns results in

$$\begin{Bmatrix} C_{L_\alpha} \\ C_{L_\delta} \end{Bmatrix} = \left[B'B\right]^{-1} \left\{B'\Delta\psi_i\right\} \tag{B12}$$

The derivatives $\left(\frac{\partial C_L}{\partial \alpha}\right)_t$, $\frac{\partial \epsilon}{\partial \alpha}$, and $\frac{\partial C_{L_t}}{\partial \delta}$ are determined from equations (A11c), (A12c), and (A7d), respectively.

Thus the derivatives $C_{L_\alpha}$, $C_{L_{\dot{\theta}}}$, $C_{L_{\dot{\alpha}}}$, $C_{L_\delta}$, $C_{m_\alpha}$, $C_{m_{\dot{\theta}}}$, $C_{m_{\dot{\alpha}}}$, $C_{m_\delta}$, $\left(\frac{\partial C_L}{\partial \alpha}\right)_t$, $\frac{\partial \epsilon}{\partial \alpha}$, and $\frac{\partial C_{L_t}}{\partial \delta}$ are now determined.

# APPENDIX C

## DETERMINATION OF LONGITUDINAL-STABILITY DERIVATIVES BY USING TWO MEASUREMENTS IN TIME-HISTORY FORM AND TWO SUPPLEMENTAL ASSUMPTIONS

By the matrix method of reference 4, time-history measurements of the pitching velocity and incremental elevator angle are used to compute the K values from the relation

$$\ddot{\theta} + K_1\dot{\theta} + K_2\,\Delta\theta = K_5\,\Delta\delta + K_6\int_0^t \Delta\delta\,dt \tag{C1}$$

where

$$K_1 = \frac{qS}{V}\left[\frac{C_{L_\alpha}}{m} - \frac{\bar{c}V}{I}\left(C_{m_{\dot{\alpha}}} + C_{m_{\dot{\theta}}}\right)\right] \tag{C2a}$$

$$K_2 = -\frac{qS\bar{c}}{I}\left(C_{m_\alpha} + C_{m_{\dot{\theta}}}\frac{C_{L_\alpha}qS}{mV}\right) \tag{C2b}$$

$$K_5 = \frac{qS\bar{c}}{I}\left(C_{m_\delta} - \frac{qS}{mV}C_{L_\delta}C_{m_{\dot{\alpha}}}\right) \tag{C2c}$$

$$K_6 = \frac{qS\bar{c}}{I}\frac{qS}{mV}\left(C_{L_\alpha}C_{m_\delta} - C_{L_\delta}C_{m_\alpha}\right) \tag{C2d}$$

If it is assumed that

$$C_{m_{\dot{\alpha}}} = \lambda C_{m_{\dot{\theta}}} \tag{C3}$$

and

$$C_{m_\delta} = \frac{x_t}{\bar{c}}C_{L_\delta} \tag{C4}$$

these six relations in six unknowns $\left(C_{L_\alpha}, C_{L_\delta}, C_{m_\alpha}, C_{m_\delta}, C_{m_{\dot\theta}}, \text{and } C_{m_{\dot\alpha}}\right)$ can now be solved simultaneously. The following relations for the variables result:

$$C_{L_\alpha} = -\frac{C_1}{2} - \sqrt{\frac{C_1^2}{4} - C_2} \tag{C5}$$

where

$$C_1 = \frac{mV}{qS}\left[(1 + \lambda)\frac{x_t V m}{I} - K_1 + \lambda\frac{K_6}{K_5}\right] \tag{C6}$$

and

$$C_2 = (1 + \lambda)\left(\frac{mV}{qS}\right)^2\left(K_2 - \frac{\lambda}{1 + \lambda}\frac{K_6}{K_5}K_1 - \frac{K_6}{K_5}\frac{x_t V m}{I}\right) \tag{C7}$$

$$C_{m_\alpha} = -\frac{I}{qS\bar{c}}K_2 - \frac{qS}{mV}\frac{I}{\bar{c}Vm(1 + \lambda)}C_{L_\alpha}^2 + \frac{K_1 I}{\bar{c}Vm(1 + \lambda)}C_{L_\alpha} \tag{C8}$$

$$C_{m_{\dot\theta}} = \frac{I}{\bar{c}Vm(1 + \lambda)}\left(C_{L_\alpha} - \frac{mV}{qS}K_1\right) \tag{C9}$$

$$C_{m_\delta} = \frac{\frac{I}{qS\bar{c}}\frac{mV}{qS\bar{c}}K_6}{\frac{C_{L_\alpha}}{\bar{c}} - \frac{C_{m_\alpha}}{x_t}} \tag{C10}$$

Approximate formulas which give a quicker evaluation of the derivatives with fair accuracy were derived from equations (C5), (C8), (C9), and (C10) and are

$$C_{L_\alpha} \approx \frac{mV}{qS} \frac{K_6}{K_5} \tag{C11}$$

$$C_{m_\delta} \approx \frac{I}{qS\bar{c}} K_5 \tag{C12}$$

$$C_{m_{\dot{\theta}}} \approx \frac{1}{1 + \lambda} \cdot \frac{I}{qS\bar{c}}\left(\frac{K_6}{K_5} - K_1\right) \tag{C13}$$

$$C_{m_\alpha} \approx - \frac{I}{qS\bar{c}} K_2 - \frac{K_6}{K_5} C_{m_{\dot{\theta}}} = \frac{I}{qS\bar{c}}\left[-K_2 - \frac{1}{1 + \lambda}\left(\frac{K_6}{K_5}\right)^2 + \frac{K_1 K_6}{(1 + \lambda)K_5}\right] \tag{C14}$$

The set of approximate formulas has been found to give results which are usually within the accuracy of the method. In table VI(c), a comparison is presented between results computed by using the approximate relations and the more accurate relations. The set of approximate formulas given by equations (C11) to (C14) is used in the development of a modified method C which is given in appendix D.

## APPENDIX D

## MODIFIED METHOD C

For some special types of longitudinal maneuvers considerable information may be determined from a single time history. If the elevator motion is known to be of the impulse type (a blip of short duration) but its magnitude or time history is unknown, then the method of appendix C may be modified slightly to yield some of the stability derivatives. The method may be used with impulse-type forcing functions produced by ballistic devices. If the input is not a pure impulse but resembles one, then the modified method may be applied after the elevator motion is zero. Integrals, however, must be evaluated from the zero-time trim condition but the least-squares procedure is applied only to the time histories after the elevator motion is zero.

Since the definite integral of an impulse is a step function and the integral of a step function is a ramp function, let

$$\int_0^t \Delta\delta \, dt = A \tag{D1}$$

and

$$\int_0^t \int_0^\tau \Delta\delta \, d\tau \, dt = At \tag{D2}$$

Substituting these values in the integral form of equation (A15) which is

$$K_1 \Delta\theta + K_2 \int_0^t \Delta\theta \, dt - K_5 \int_0^t \Delta\delta \, dt - K_6 \int_0^t \int_0^\tau \Delta\delta \, d\tau \, dt = -\dot{\theta} \tag{D3}$$

results in

$$K_1 \Delta\theta + K_2 \int_0^t \Delta\theta \, dt - K_5 A - K_6 At = -\dot{\theta} \tag{D4}$$

Equation (D4) may be expressed in matrix form as

$$\begin{Vmatrix} \Delta\theta_0 & \int_0^{t_0} \Delta\theta\, dt & -1 & -t_0 \\ \Delta\theta_1 & \int_0^{t_1} \Delta\theta\, dt & -1 & -t_1 \\ \Delta\theta_2 & \int_0^{t_2} \Delta\theta\, dt & -1 & -t_2 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \Delta\theta_n & \int_0^{t_n} \Delta\theta\, dt & -1 & -t_n \end{Vmatrix} \begin{Bmatrix} K_1 \\ K_2 \\ K_5A \\ K_6A \end{Bmatrix} = \begin{Bmatrix} -\dot{\theta}_0 \\ -\dot{\theta}_1 \\ -\dot{\theta}_2 \\ \cdot \\ \cdot \\ \cdot \\ -\dot{\theta}_n \end{Bmatrix} \qquad \text{(D5)}$$

Equation (D5) is then used to solve for the stability coefficients $K_1$, $K_2$, $K_5A$, and $K_6A$.

The following approximate formulas presented in appendix C are used to compute the stability derivatives (since only the ratio $K_6/K_5$ is used the value of $A$ need not be evaluated):

$$C_{L_\alpha} \approx \frac{mV}{qS}\frac{K_6}{K_5} \qquad \text{(D6)}$$

$$C_{m_{\dot{\theta}}} \approx \frac{1}{1 + \lambda}\frac{I}{qS\bar{c}}\left(\frac{K_6}{K_5} - K_1\right) \qquad \text{(D7)}$$

$$C_{m_\alpha} \approx -\frac{I}{qS\bar{c}}K_2 - \frac{K_6}{K_5}C_{m_{\dot{\theta}}} = \frac{I}{qS\bar{c}}\left[-K_2 - \frac{1}{1 + \lambda}\left(\frac{K_6}{K_5}\right)^2 + \frac{K_1K_6}{(1 + \lambda)K_5}\right] \qquad \text{(D8)}$$

Also, it is assumed that

$$C_{m_{\dot{\alpha}}} = \lambda C_{m_{\dot{\theta}}} \tag{D9}$$

As indicated previously in appendix A,

$$C_{L_{\dot{\alpha}}} = \frac{\bar{c}}{x_t} C_{m_{\dot{\alpha}}} \tag{D10}$$

$$C_{L_{\dot{\theta}}} = \frac{\bar{c}}{x_t} C_{m_{\dot{\theta}}} \tag{D11}$$

Thus the analysis of a single time history of pitching velocity can yield considerable information if it is the response to an elevator impulse function; however, the elevator-effectiveness derivatives cannot be found by this method.

# APPENDIX E

## DEFINITIONS OF STABILITY PARAMETERS

The stability parameters of the methods presented, not previously defined in the original list of symbols, can be defined as follows:

$$C_1 = \frac{mV}{qS}\left[\lambda \frac{K_6}{K_5} + (1 + \lambda)\frac{Vx_t m}{I} - K_1\right]$$

$$C_2 = (1 + \lambda)\left(\frac{mV}{qS}\right)^2\left(K_2 - \frac{\lambda}{1 + \lambda}\frac{K_6}{K_5} K_1 - \frac{K_6}{K_5}\frac{x_t Vm}{I}\right)$$

$$C_{L_{\dot{\alpha}}} = -\left(\frac{\partial C_L}{\partial \alpha}\right)_t \eta_t \frac{S_t}{S}\frac{x_t}{V}\frac{\partial \epsilon}{\partial \alpha}$$

$$C_{L_\alpha} = \left(\frac{\partial C_L}{\partial \alpha}\right)_{WB} + \left(\frac{\partial C_L}{\partial \alpha}\right)_t \eta_t \frac{S_t}{S}\left(1 - \frac{\partial \epsilon}{\partial \alpha}\right)$$

$$C_{L_{\dot{\theta}}} = -\left(\frac{\partial C_L}{\partial \alpha}\right)_t \eta_t \frac{S_t}{S}\frac{x_t}{V}\frac{1}{\sqrt{\eta_t}}$$

$$C_{L_\delta} = \frac{\partial C_{L_t}}{\partial \delta}\eta_t \frac{S_t}{S}$$

$$C_{m_\alpha} = \left(\frac{\partial C_m}{\partial \alpha}\right)_{WB} + \left(\frac{\partial C_L}{\partial \alpha}\right)_t \eta_t \frac{S_t}{S}\frac{x_t}{\bar{c}}\left(1 - \frac{\partial \epsilon}{\partial \alpha}\right)$$

$$C_{m_{\dot{\alpha}}} = -\left(\frac{\partial C_L}{\partial \alpha}\right)_t \eta_t \frac{S_t}{S} \frac{x_t^2}{\bar{c}V} \frac{\partial \epsilon}{\partial \alpha}$$

$$C_{m_{\dot{\theta}}} = -\left(\frac{\partial C_L}{\partial \alpha}\right)_t \eta_t \frac{S_t}{S} \frac{x_t^2}{\bar{c}V} \frac{1}{\sqrt{\eta_t}}$$

$$C_{m_\delta} = \frac{\partial C_{L_t}}{\partial \delta} \eta_t \frac{S_t}{S} \frac{x_t}{\bar{c}} + \frac{\partial C_{m_t}}{\partial \delta} \eta_t \frac{S_t}{S} \frac{\bar{c}_t}{\bar{c}}$$

$$K_1 = \frac{qS}{V}\left[\frac{C_{L_\alpha}}{m} - \frac{\bar{c}V}{I}\left(C_{m_{\dot{\alpha}}} + C_{m_{\dot{\theta}}}\right)\right]$$

$$K_2 = -\frac{qS\bar{c}}{I}\left(C_{m_\alpha} + C_{m_{\dot{\theta}}} \frac{C_{L_\alpha} qS}{mV}\right)$$

$$K_5 = \frac{qS\bar{c}}{I}\left(C_{m_\delta} - \frac{qS}{mV} C_{L_\delta} C_{m_{\dot{\alpha}}}\right)$$

$$K_6 = \frac{qS\bar{c}}{I} \frac{qS}{mV}\left(C_{L_\alpha} C_{m_\delta} - C_{L_\delta} C_{m_\alpha}\right)$$

$$K_{10} = \frac{I}{\bar{c}Vm} C_{L_\alpha} - \frac{I}{qS\bar{c}} K_1$$

$$\Delta\mu = \frac{\Delta L_t}{qS} - \frac{\bar{c}V}{x_t^2} K_{10} \Delta\alpha - \frac{\bar{c}}{x_t} K_{10}\dot{\alpha} - C_{L_\delta} \Delta\delta$$

$$\Delta\xi = (1 + \lambda)\dot{\theta} - \lambda \frac{g}{V} \Delta n$$

$$\Delta\sigma = \frac{I}{qS\bar{c}} \dot{\theta} - C_{m_{\dot{\alpha}}} \Delta\alpha - C_{m_{\dot{\theta}}} \Delta\theta$$

$$\Delta\varphi = \frac{\bar{c}}{x_t}\left[\frac{g}{V} \Delta n - \frac{V}{x_t}\left(\sqrt{\eta_t} + 1\right)\Delta\alpha\right]$$

$$\Delta\psi = \frac{W}{qS} \Delta n - C_{L_{\dot{\theta}}}\dot{\theta} - C_{L_{\dot{\alpha}}}\dot{\alpha}$$

$$\bar{K}_1 = \frac{qS\bar{c}}{I}\left[\frac{I}{qS\bar{c}} \frac{C_{L_\alpha}}{\frac{mV}{qS} + C_{L_{\dot{\alpha}}}} - C_{m_{\dot{\alpha}}} \frac{\frac{mV}{qS} - C_{L_{\dot{\theta}}}}{\frac{mV}{qS} + C_{L_{\dot{\alpha}}}} - C_{m_{\dot{\theta}}}\right]$$

$$\bar{K}_2 = \frac{-C_{m_\alpha} - \frac{qS}{mV}\left(C_{m_{\dot{\theta}}}C_{L_\alpha} - C_{L_{\dot{\theta}}}C_{m_\alpha}\right)}{\frac{I}{qS\bar{c}}\left(1 + \frac{qS}{mV} C_{L_{\dot{\alpha}}}\right)}$$

$$\bar{K}_3 = \frac{qS\bar{c}}{I} \frac{C_{m_{\dot{\theta}}}C_{L_\delta} + C_{m_\delta}\left(\frac{mV}{qS} - C_{L_{\dot{\theta}}}\right)}{\frac{mV}{qS} + C_{L_{\dot{\alpha}}}}$$

$$\overline{K}_4 = -\frac{C_{L_\delta}}{\frac{mV}{qS} + C_{L_{\dot{\alpha}}}}$$

$$\overline{K}_5 = \frac{qS\overline{c}\left(C_{L_{\dot{\alpha}}}C_{m_\delta} - C_{m_{\dot{\alpha}}}C_{L_\delta} + \frac{mV}{qS}C_{m_\delta}\right)}{I\left(\frac{mV}{qS} + C_{L_{\dot{\alpha}}}\right)}$$

$$\overline{K}_6 = \frac{qS\overline{c}\left(C_{L_\alpha}C_{m_\delta} - C_{L_\delta}C_{m_\alpha}\right)}{I\left(\frac{mV}{qS} + C_{L_{\dot{\alpha}}}\right)}$$

Matrices used in the present paper are defined as follows:

$$\|B\| = \begin{Vmatrix} \Delta\alpha_0 & \Delta\delta_0 \\ \Delta\alpha_1 & \Delta\delta_1 \\ \Delta\alpha_2 & \Delta\delta_2 \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \Delta\alpha_n & \Delta\delta_n \end{Vmatrix}$$

$\|C\|$ is the integrating matrix given in table I.

$$\|D\| = \left\| \begin{matrix} \int_0^{t_0} \Delta\alpha \, dt & \int_0^{t_0} \Delta\delta \, dt \\ \int_0^{t_1} \Delta\alpha \, dt & \int_0^{t_1} \Delta\delta \, dt \\ \int_0^{t_2} \Delta\alpha \, dt & \int_0^{t_2} \Delta\delta \, dt \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \int_0^{t_n} \Delta\alpha \, dt & \int_0^{t_n} \Delta\delta \, dt \end{matrix} \right\|$$

$$\|E\| = \left\| \begin{matrix} \int_0^{t_0} \Delta\alpha \, dt & \int_0^{t_0} \Delta\xi \, dt & \int_0^{t_0} \Delta\delta \, dt \\ \int_0^{t_1} \Delta\alpha \, dt & \int_0^{t_1} \Delta\xi \, dt & \int_0^{t_1} \Delta\delta \, dt \\ \int_0^{t_2} \Delta\alpha \, dt & \int_0^{t_2} \Delta\xi \, dt & \int_0^{t_2} \Delta\delta \, dt \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \int_0^{t_n} \Delta\alpha \, dt & \int_0^{t_n} \Delta\xi \, dt & \int_0^{t_n} \Delta\delta \, dt \end{matrix} \right\|$$

## REFERENCES

1. Milliken, W. F., Jr.: Dynamic Stability and Control Research. Rep. No. CAL-39, Cornell Aero. Lab., Inc. (Presented at Third International Joint Conference of the R.A.S. - I.A.S., Brighton, England, Sept. 3-14, 1951.)

2. Greenberg, Harry: A Survey of Methods for Determining Stability Parameters of an Airplane From Dynamic Flight Measurements. NACA TN 2340, 1951.

3. Diederich, Franklin W.: Calculation of the Aerodynamic Loading of Swept and Unswept Flexible Wings of Arbitrary Stiffness. NACA Rep. 1000, 1950. (Supersedes NACA TN 1876.)

4. Donegan, James J., and Pearson, Henry A.: Matrix Method of Determining the Longitudinal-Stability Coefficients and Frequency Response of an Aircraft From Transient Flight Data. NACA Rep. 1070, 1952. (Supersedes NACA TN 2370.)

5. Jones, B. Melvill: Dynamics of the Airplane. Numerical Solution of the Symmetric Equations of Chapter V. Vol. V of Aerodynamic Theory, div. N, ch. VI, sec. 39, W. F. Durand, ed., Julius Springer (Berlin), 1935, p. 175.

6. Scanlan, Robert H., and Rosenbaum, Robert: Introduction to the Study of Aircraft Vibration and Flutter. The Macmillan Co., 1951.

7. Pearson, Henry A.: Derivation of Charts for Determining the Horizontal Tail Load Variation With Any Elevator Motion. NACA Rep. 759, 1943. (Supersedes NACA WR L-488.)

TABLE I.- INTEGRATING MATRIX $||C||$

$$
||C|| = \frac{\Delta t}{12}\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdot & \cdot & \cdot \\
5 & 8 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdot & \cdot & \cdot \\
4 & 16 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdot & \cdot & \cdot \\
4 & 16 & 9 & 8 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdot & \cdot & \cdot \\
4 & 16 & 8 & 16 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdot & \cdot & \cdot \\
4 & 16 & 8 & 16 & 9 & 8 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdot & \cdot & \cdot \\
4 & 16 & 8 & 16 & 8 & 16 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdot & \cdot & \cdot \\
4 & 16 & 8 & 16 & 8 & 16 & 9 & 8 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & \cdot & \cdot & \cdot \\
4 & 16 & 8 & 16 & 8 & 16 & 8 & 16 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & \cdot & \cdot & \cdot \\
4 & 16 & 8 & 16 & 8 & 16 & 8 & 16 & 9 & 8 & -1 & 0 & 0 & 0 & 0 & \cdot & \cdot & \cdot \\
4 & 16 & 8 & 16 & 8 & 16 & 8 & 16 & 8 & 16 & 4 & 0 & 0 & 0 & 0 & \cdot & \cdot & \cdot \\
4 & 16 & 8 & 16 & 8 & 16 & 8 & 16 & 8 & 16 & 9 & 8 & -1 & 0 & 0 & \cdot & \cdot & \cdot \\
4 & 16 & 8 & 16 & 8 & 16 & 8 & 16 & 8 & 16 & 8 & 16 & 4 & 0 & 0 & \cdot & \cdot & \cdot \\
4 & 16 & 8 & 16 & 8 & 16 & 8 & 16 & 8 & 16 & 8 & 16 & 9 & 8 & -1 & \cdot & \cdot & \cdot \\
4 & 16 & 8 & 16 & 8 & 16 & 8 & 16 & 8 & 16 & 8 & 16 & 8 & 16 & 4 & \cdot & \cdot & \cdot \\
4 & 16 & 8 & 16 & 8 & 16 & 8 & 16 & 8 & 16 & 8 & 16 & 8 & 16 & 9 & \cdot & \cdot & \cdot \\
4 & 16 & 8 & 16 & 8 & 16 & 8 & 16 & 8 & 16 & 8 & 16 & 8 & 16 & 8 & \cdot & \cdot & \cdot \\
4 & 16 & 8 & 16 & 8 & 16 & 8 & 16 & 8 & 16 & 8 & 16 & 8 & 16 & 8 & \cdot & \cdot & \cdot \\
4 & 16 & 8 & 16 & 8 & 16 & 8 & 16 & 8 & 16 & 8 & 16 & 8 & 16 & 8 & \cdot & \cdot & \cdot \\
4 & 16 & 8 & 16 & 8 & 16 & 8 & 16 & 8 & 16 & 8 & 16 & 8 & 16 & 8 & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot
\end{bmatrix}
$$

TABLE II.- AIRPLANE CHARACTERISTICS, FLIGHT CONDITIONS,

AND TRANSFER-FUNCTION COEFFICIENTS

(a) Airplane characteristics and flight conditions

| | Flight 1 | Flight 2 | Flight 3 |
|---|---|---|---|
| $b$, ft | 89 | 89 | 89 |
| $\bar{c}$, ft | 14.016 | 14.016 | 14.016 |
| Center-of-gravity position, percent M.A.C. | 27.34 | 27.32 | 27.44 |
| $g/V$, 1/sec | 0.061923 | 0.062854 | 0.062686 |
| $I$, slug-ft | 255,865 | 255,276 | 258,957 |
| $k_y^2$, ft$^2$ | 141.61 | 141.61 | 141.61 |
| Mach number | 0.497 | 0.494 | 0.496 |
| $m$, slugs | 1806.83 | 1802.67 | 1828.66 |
| $q$, lb/ft$^2$ | 171 | 166 | 171 |
| $S$, ft$^2$ | 1,175 | 1,175 | 1,175 |
| $S_t$, ft$^2$ | 289.3 | 289.3 | 289.3 |
| $V$, ft/sec | 520 | 512 | 514 |
| $W$, lb | 58,180 | 58,050 | 58,880 |
| $W/qS$ | 0.289561 | 0.297595 | 0.293060 |
| $x_t$, ft | -33.5 | -33.5 | -33.5 |
| $\eta_t$ | 0.87 | 0.87 | 0.87 |
| $\rho$, slugs/ft$^3$ | 0.001267 | 0.001276 | 0.001281 |

(b) Coefficients of airplane transfer function

| | Flight 1 | | Flight 2 | |
|---|---|---|---|---|
| | Coefficient | Probable error | Coefficient | Probable error |
| $K_1$ | 4.14 | 0.14 | 4.19 | 0.18 |
| $K_2$ | 9.547 | 0.73 | 10.329 | 0.70 |
| $K_5$ | -9.767 | 0.35 | -10.010 | 0.42 |
| $K_6$ | -14.624 | 1.4 | -15.526 | 1.3 |

TABLE III.- TIME HISTORIES OF MEASURED AND DERIVED

QUANTITIES FOR FLIGHT 1

| Measured | | | | | Derived | | | | |
|---|---|---|---|---|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) |
| $t$ | $\Delta\delta$ | $\Delta L_t$ | $\Delta n$ | $\dot{\theta}$ | $\Delta\alpha$ | $\Delta\xi$ | $\Delta\varphi$ | $\Delta\mu$ | $\Delta\sigma$ |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| .1 | .009703 | 660 | 0 | 0 | 0 | 0 | 0 | .000741 | 0 |
| .2 | .055812 | 3773 | .03792 | -.019808 | -.000922 | -.030886 | .010591 | .004929 | -.001986 |
| .3 | .072880 | 3488 | -.12008 | -.074064 | -.005175 | -.107378 | .068067 | -.003122 | -.007957 |
| .4 | .074625 | 1644 | -.35392 | -.125305 | -.013921 | -.177000 | .183905 | -.022667 | -.014935 |
| .5 | .071414 | 25 | -.62568 | -.149419 | -.024871 | -.204757 | .328389 | -.046165 | -.020224 |
| .6 | .070698 | -1423 | -.92272 | -.165781 | -.035932 | -.220103 | .474922 | -.070393 | -.025122 |
| .7 | .067923 | -2529 | -1.16920 | -.167504 | -.046216 | -.215056 | .610391 | -.092683 | -.028773 |
| .8 | .068691 | -3459 | -1.40936 | -.163628 | -.054782 | -.201806 | .724134 | -.112697 | -.031746 |
| .9 | .067923 | -3962 | -1.56736 | -.155447 | -.061513 | -.184643 | .812714 | -.127257 | -.034084 |
| 1.0 | .064712 | -4518 | -1.74432 | -.142960 | -.066176 | -.160433 | .875829 | -.138772 | -.035700 |
| 1.1 | .043246 | -6321 | -1.85808 | -.127027 | -.068518 | -.133012 | .908173 | -.148055 | -.036590 |
| 1.2 | .032077 | -6274 | -1.75064 | -.090426 | -.068301 | -.081436 | .902665 | -.147554 | -.034981 |
| 1.3 | -.012565 | -8570 | -1.75064 | -.053810 | -.064585 | -.026512 | .856022 | -.145166 | -.032478 |
| 1.4 | -.022862 | -6778 | -1.45360 | .014640 | -.056328 | .066966 | .744686 | -.125729 | -.025852 |
| 1.5 | -.026023 | -5079 | -1.25768 | .058562 | -.044150 | .126783 | .586752 | -.100705 | -.020274 |
| 1.6 | -.047260 | -4748 | -.88480 | .091287 | -.030073 | .164326 | .400397 | -.072582 | -.014992 |
| 1.7 | -.075428 | -4048 | -.46136 | .133055 | -.014737 | .213867 | .196931 | -.039441 | -.008194 |
| 1.8 | -.071623 | -1968 | -.05056 | .167503 | .001884 | .252820 | -.022338 | -.005166 | -.001334 |
| 1.9 | -.078674 | -316 | .28440 | .196353 | .019360 | .285725 | -.250374 | .031245 | .005589 |
| 2.0 | -.082513 | 1018 | .72680 | .207549 | .036467 | .288821 | -.476562 | .067093 | .011195 |
| 2.1 | -.088063 | 2172 | 1.17552 | .214008 | .051697 | .284616 | -.679353 | .100087 | .016361 |
| 2.2 | -.095707 | 2835 | 1.46624 | .217022 | .065082 | .280136 | -.854892 | .127688 | .021140 |
| 2.3 | -.087504 | 5070 | 1.73168 | .217022 | .076895 | .271918 | -1.010045 | .156658 | .025542 |
| 2.4 | -.030227 | 9801 | 2.07296 | .186880 | .085314 | .216138 | -1.124562 | .183397 | .026734 |
| 2.5 | -.024956 | | 1.85808 | .121859 | .088583 | .125260 | | | |
| 2.6 | -.026422 | | 1.78224 | .051241 | .085705 | .021681 | | | |
| 2.7 | -.026702 | | 1.62424 | .006889 | .077804 | -.039955 | | | |
| 2.8 | -.019651 | | 1.46624 | -.039516 | .066452 | -.104671 | | | |
| 2.9 | -.005864 | | 1.22608 | -.072772 | .052350 | -.147119 | | | |
| 3.0 | .000908 | | .99856 | -.080953 | .037762 | -.152347 | | | |
| 3.1 | .005620 | | .75208 | -.088704 | .023846 | -.156342 | | | |
| 3.2 | .007958 | | .34760 | -.083537 | .012066 | -.136064 | | | |
| 3.3 | .010611 | | .29072 | -.085259 | .001887 | -.136890 | | | |
| 3.4 | .009145 | | .01264 | -.068896 | -.006638 | -.103735 | | | |
| 3.5 | .010175 | | -.18960 | -.062437 | -.012536 | -.087786 | | | |

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TABLE IV.- COMPUTATIONS ILLUSTRATING METHOD A

(a) First approximation of $C_{L_\alpha}$ and $C_{L_\delta}$ by step (3)

| t | $\Delta\alpha$ (table III, column 6) | $\Delta\delta$ (table III, column 2) | $\frac{W}{qS}\Delta n$ | t | $\Delta\alpha$ (table III, column 6) | $\Delta\delta$ (table III, column 2) | $\frac{W}{qS}\Delta n$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1.8 | 0.001884 | -0.071623 | -0.014640 |
| .1 | 0 | .009703 | 0 | 1.9 | .019360 | -.078674 | .082351 |
| .2 | -.000922 | .055812 | .010980 | 2.0 | .036467 | -.082513 | .210453 |
| .3 | -.005175 | .072880 | -.034770 | 2.1 | .051697 | -.088063 | .340384 |
| .4 | -.013921 | .074625 | -.102481 | 2.2 | .065082 | -.095707 | .424566 |
| .5 | -.024871 | .071414 | -.181172 | 2.3 | .076895 | -.087504 | .501427 |
| .6 | -.035932 | .070698 | -.267184 | 2.4 | .085314 | -.030227 | .600248 |
| .7 | -.046216 | .067923 | -.338554 | 2.5 | .088583 | -.024956 | .538027 |
| .8 | -.054782 | .068691 | -.408095 | 2.6 | .085705 | -.026422 | .516067 |
| .9 | -.061513 | .067923 | -.453846 | 2.7 | .077804 | -.026702 | .470316 |
| 1.0 | -.066176 | .064712 | -.505087 | 2.8 | .066452 | -.019651 | .424566 |
| 1.1 | -.068518 | .043246 | -.538027 | 2.9 | .052350 | -.005864 | .355025 |
| 1.2 | -.068301 | .032077 | -.506917 | 3.0 | .037762 | .000908 | .289144 |
| 1.3 | -.064585 | -.012565 | -.506917 | 3.1 | .023846 | .005620 | .217773 |
| 1.4 | -.056328 | -.022862 | -.420906 | 3.2 | .012066 | .007958 | .100651 |
| 1.5 | -.044150 | -.026023 | -.364175 | 3.3 | .001887 | .010611 | .084181 |
| 1.6 | -.030073 | -.047260 | -.256203 | 3.4 | -.006638 | .009145 | .003660 |
| 1.7 | -.014737 | -.075428 | -.133592 | 3.5 | -.012536 | .010175 | -.054901 |

$$C_{L_\alpha}\,\Delta\alpha + C_{L_\delta}\,\Delta\delta = \frac{W}{qS}\,\Delta n$$

$$\begin{bmatrix} 0.087761 & -0.053345 \\ -0.053345 & 0.100964 \end{bmatrix} \begin{Bmatrix} C_{L_\alpha} \\ C_{L_\delta} \end{Bmatrix} = \begin{bmatrix} 0.606401 \\ -0.350626 \end{bmatrix}$$

$$C_{L_\alpha} = 7.07$$

$$C_{L_\delta} = 0.262$$

TABLE IV.- COMPUTATIONS ILLUSTRATING METHOD A - Continued

(b) Determination of $C_{m_{\dot{\theta}}}$ amd $C_{m_{\dot{\alpha}}}$ by steps (7) and (8)

| t | Δφ (table III, column 8) | Δμ (table III, column 9) | t | Δφ (table III, column 8) | Δμ (table III, column 9) |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 1.2 | 0.902665 | -0.147554 |
| .1 | 0 | .000741 | 1.3 | .856022 | -.145166 |
| .2 | .010591 | .004929 | 1.4 | .744686 | -.125729 |
| .3 | .068067 | -.003122 | 1.5 | .586752 | -.100705 |
| .4 | .183905 | -.022667 | 1.6 | .400397 | -.072582 |
| .5 | .328389 | -.046165 | 1.7 | .196931 | -.039441 |
| .6 | .474922 | -.070393 | 1.8 | -.022338 | -.005166 |
| .7 | .610391 | -.092683 | 1.9 | -.250374 | .031245 |
| .8 | .724134 | -.112697 | 2.0 | -.476562 | .067093 |
| .9 | .812714 | -.127257 | 2.1 | -.679353 | .100087 |
| 1.0 | .875829 | -.138772 | 2.2 | -.854892 | .127688 |
| 1.1 | .908173 | -.148055 | 2.3 | -1.010045 | .156658 |
| | | | 2.4 | -1.124562 | .183397 |

$$C_{m_{\dot{\theta}}} \left\{\Delta\varphi_i\right\} = \left\{\Delta\mu_i\right\}$$

$$9.934264 C_{m_{\dot{\theta}}} = -1.578836$$

$$C_{m_{\dot{\theta}}} = -0.1589$$

$$C_{m_{\dot{\alpha}}} = K_{10} - C_{m_{\dot{\theta}}}$$

$$C_{m_{\dot{\alpha}}} = -0.0803$$

TABLE IV.- COMPUTATIONS ILLUSTRATING METHOD A.- Continued

(c) Determination of $C_{m_\alpha}$ and $C_{m_\delta}$ by step (10)

| $t$ | $\int_0^t \Delta\alpha\, dt$ | $\int_0^t \Delta\delta\, dt$ | $\Delta\sigma$ | $t$ | $\int_0^t \Delta\alpha\, dt$ | $\int_0^t \Delta\delta\, dt$ | $\Delta\sigma$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1.2 | -0.041237 | 0.067784 | -0.034981 |
| .1 | 0 | .000182 | 0 | 1.3 | -.047910 | .068473 | -.032478 |
| .2 | -.000018 | .003154 | -.001986 | 1.4 | -.053989 | .066415 | -.025852 |
| .3 | -.000295 | .009716 | -.007957 | 1.5 | -.059045 | .064119 | -.020274 |
| .4 | -.001232 | .017219 | -.014935 | 1.6 | -.062767 | .060601 | -.014992 |
| .5 | -.003153 | .024501 | -.020224 | 1.7 | -.065018 | .054201 | -.008194 |
| .6 | -.006200 | .031585 | -.025122 | 1.8 | -.065668 | .046582 | -.001334 |
| .7 | -.010314 | .038487 | -.028773 | 1.9 | -.064612 | .039040 | .005589 |
| .8 | -.015379 | .045288 | -.031746 | 2.0 | -.061805 | .030954 | .011195 |
| .9 | -.021209 | .052139 | -.034084 | 2.1 | -.057382 | .022443 | .016361 |
| 1.0 | -.027613 | .058791 | -.035700 | 2.2 | -.051530 | .013272 | .021140 |
| 1.1 | -.034367 | .064103 | -.036590 | 2.3 | -.044418 | .003702 | .025542 |
| | | | | 2.4 | -.036264 | -.002593 | .026734 |

$$\Delta\sigma = C_{m_\alpha}\int_0^t \Delta\alpha\, dt + C_{m_\delta}\int_0^t \Delta\delta\, dt$$

$$\begin{bmatrix} 0.042893 & -0.036635 \\ -0.036635 & 0.045583 \end{bmatrix} \begin{Bmatrix} C_{m_\alpha} \\ C_{m_\delta} \end{Bmatrix} = \begin{Bmatrix} 0.005959 \\ -0.018185 \end{Bmatrix}$$

$$C_{m_\alpha} = -0.644$$

$$C_{m_\delta} = -0.916$$

TABLE IV.- COMPUTATIONS ILLUSTRATING METHOD A - Continued

(d) Determination of the refined values of $C_{L_\alpha}$ and $C_{L_\delta}$ by step (13)

| t | $\Delta\alpha$ (table III, column 6) | $\Delta\delta$ (table III, column 2) | $\Delta\psi$ | t | $\Delta\alpha$ (table III, column 6) | $\Delta\delta$ (table III, column 2) | $\Delta\psi$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1.8 | 0.001884 | -0.071623 | -0.031511 |
| .1 | 0 | .009703 | 0 | 1.9 | .019360 | -.078674 | .063290 |
| .2 | -.000922 | .055812 | .013041 | 2.0 | .036467 | -.082513 | .191190 |
| .3 | -.005175 | .072880 | -.027606 | 2.1 | .051697 | -.088063 | .321409 |
| .4 | -.013921 | .074625 | -.090675 | 2.2 | .065082 | -.095707 | .405894 |
| .5 | -.024871 | .071414 | -.167519 | 2.3 | .076895 | -.087504 | .483307 |
| .6 | -.035932 | .070698 | -.252511 | 2.4 | .085314 | -.030227 | .585856 |
| .7 | -.046216 | .067923 | -.324221 | 2.5 | .088583 | -.024956 | .529696 |
| .8 | -.054782 | .068691 | -.394650 | 2.6 | .085705 | -.026422 | .514646 |
| .9 | -.061513 | .067923 | -.441549 | 2.7 | .077804 | -.026702 | .473006 |
| 1.0 | -.066176 | .064712 | -.494407 | 2.8 | .066452 | -.019651 | .431572 |
| 1.1 | -.068518 | .043246 | -.529178 | 2.9 | .052350 | -.005864 | .364860 |
| 1.2 | -.068301 | .032077 | -.501508 | 3.0 | .037762 | .000908 | .299324 |
| 1.3 | -.064585 | -.012565 | -.505173 | 3.1 | .023846 | .005620 | .228216 |
| 1.4 | -.056328 | -.022862 | -.425395 | 3.2 | .012066 | .007958 | .109736 |
| 1.5 | -.044150 | -.026023 | -.372653 | 3.3 | .001887 | .010611 | .093319 |
| 1.6 | -.030073 | -.047260 | -.267181 | 3.4 | -.006638 | .009145 | .010582 |
| 1.7 | -.014737 | -.075428 | -.147869 | 3.5 | -.012536 | .010175 | -.049055 |

$$\Delta\psi = C_{L_\alpha}\,\Delta\alpha + C_{L_\delta}\,\Delta\delta$$

$$\begin{bmatrix} 0.087761 & -0.053345 \\ -0.053345 & 0.100964 \end{bmatrix} \begin{Bmatrix} C_{L_\alpha} \\ C_{L_\delta} \end{Bmatrix} = \begin{Bmatrix} 0.597599 \\ -0.331104 \end{Bmatrix}$$

$$C_{L_\alpha} = 7.09$$

$$C_{L_\delta} = 0.469$$

TABLE IV.- COMPUTATIONS ILLUSTRATING METHOD A - Concluded

(e) Final results using method A after three iterations

| | Flight 1 | Probable error for flight 1 | Flight 2 |
|---|---|---|---|
| $C_{L_\alpha}$ | 7.09 | 0.113 | 6.70 |
| $C_{L_\delta}$ | 0.468 | 0.105 | 0.446 |
| $C_{L_{\dot{\theta}}}$ | 0.072 | 0.0004 | 0.076 |
| $C_{L_{\dot{\alpha}}}$ | 0.028 | 0.0004 | 0.033 |
| $C_{m_\alpha}$ | -0.622 | 0.003 | -0.711 |
| $C_{m_{\dot{\theta}}}$ | -0.171 | 0.001 | -0.181 |
| $C_{m_{\dot{\alpha}}}$ | -0.068 | 0.001 | -0.078 |
| $C_{m_\delta}$ | -0.914 | 0.003 | -0.968 |
| $(\partial C_L/\partial\alpha)_t$ | 4.84 | | 5.05 |
| $\partial\epsilon/\partial\alpha$ | 0.425 | | 0.461 |
| $\partial C_{L_t}/\partial\delta$ | 2.19 | | 2.08 |

TABLE V.- COMPUTATIONS ILLUSTRATING METHOD B

(a) Determination of $C_{m_\alpha}$, $C_{m_{\dot\theta}}$, and $C_{m_\delta}$

| $t$ | $\int_0^t \Delta\alpha\ dt$ | $\int_0^t \Delta\xi\ dt$ | $\int_0^t \Delta\delta\ dt$ | $\frac{I}{qS\bar{c}}\dot\theta$ | $t$ | $\int_0^t \Delta\alpha\ dt$ | $\int_0^t \Delta\xi\ dt$ | $\int_0^t \Delta\delta\ dt$ | $\frac{I}{qS\bar{c}}\dot\theta$ |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 1.8 | -0.065668 | -0.104508 | 0.046582 | 0.015219 |
| .1 | 0 | 0 | .000182 | 0 | 1.9 | -.064612 | -.077530 | .039040 | .017840 |
| .2 | -.000018 | -.001164 | .003154 | -.001800 | 2.0 | -.061805 | -.048742 | .030954 | .018857 |
| .3 | -.000295 | -.007697 | .009716 | -.006729 | 2.1 | -.057382 | -.020009 | .022443 | .019444 |
| .4 | -.001232 | -.022265 | .017219 | -.011385 | 2.2 | -.051530 | .008260 | .013272 | .019718 |
| .5 | -.003153 | -.041702 | .024501 | -.013576 | 2.3 | -.044418 | .035893 | .003702 | .019718 |
| .6 | -.006200 | -.063115 | .031585 | -.015062 | 2.4 | -.036264 | .060589 | -.002593 | .016979 |
| .7 | -.010314 | -.085043 | .038487 | -.015219 | 2.5 | -.027526 | .077951 | -.005296 | .011072 |
| .8 | -.015379 | -.105918 | .045288 | -.014867 | 2.6 | -.018770 | .084949 | -.007809 | .004656 |
| .9 | -.021209 | -.125274 | .052139 | -.014123 | 2.7 | -.010553 | .083685 | -.010526 | .000626 |
| 1.0 | -.027613 | -.142554 | .058791 | -.012989 | 2.8 | -.003317 | .076269 | -.012905 | -.003590 |
| 1.1 | -.034367 | -.157253 | .064103 | -.011541 | 2.9 | .002646 | .063493 | -.014122 | -.006612 |
| 1.2 | -.041237 | -.168003 | .067784 | -.008216 | 3.0 | .007146 | .048510 | -.014312 | -.007355 |
| 1.3 | -.047910 | -.173429 | .068473 | -.004889 | 3.1 | .010221 | .033065 | -.013966 | -.008059 |
| 1.4 | -.053989 | -.171125 | .066415 | .001330 | 3.2 | .012003 | .018620 | -.013267 | -.007590 |
| 1.5 | -.059045 | -.161157 | .064119 | .005321 | 3.3 | .012687 | .005148 | -.012304 | -.007746 |
| 1.6 | -.062767 | -.146702 | .060601 | .008294 | 3.4 | .012428 | -.006739 | -.011282 | -.000280 |
| 1.7 | -.065018 | -.127892 | .054201 | .012089 | 3.5 | .011447 | -.016172 | -.010294 | -.000869 |

$$\frac{I}{qS\bar{c}}\dot\theta = C_{m_\alpha}\int_0^t \Delta\alpha\ dt + C_{m_{\dot\theta}}\int_0^t \Delta\xi\ dt + C_{m_\delta}\int_0^t \Delta\delta\ dt$$

$$\begin{bmatrix} 0.0448786 & 0.0743828 & -0.0370442 \\ 0.0743828 & 0.2959167 & -0.1121529 \\ -0.0370442 & -0.1121529 & 0.0471095 \end{bmatrix} \begin{Bmatrix} C_{m_\alpha} \\ C_{m_{\dot\theta}} \\ C_{m_\delta} \end{Bmatrix} = \begin{Bmatrix} -0.0072099 \\ 0.0059555 \\ -0.0006984 \end{Bmatrix}$$

$$C_{m_\alpha} = -0.624$$

$$C_{m_{\dot\theta}} = -0.149$$

$$C_{m_\delta} = -0.861$$

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TABLE V.- COMPUTATIONS ILLUSTRATING METHOD B - Continued

(b) Determination of $C_{L_\alpha}$ and $C_{L_\delta}$

| t | $\Delta\alpha$ (table III, column ⑥) | $\Delta\delta$ (table III, column ②) | $\Delta\psi$ | t | $\Delta\alpha$ (table III, column ⑥) | $\Delta\delta$ (table III, column ②) | $\Delta\psi$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1.8 | 0.001884 | -0.071623 | -0.030434 |
| .1 | 0 | .009703 | 0 | 1.9 | .019360 | -.078674 | .064502 |
| .2 | -.000922 | .055812 | .012909 | 2.0 | .036467 | -.082513 | .192411 |
| .3 | -.005175 | .072880 | -.028063 | 2.1 | .051697 | -.088063 | .322604 |
| .4 | -.013921 | .074625 | -.091424 | 2.2 | .065082 | -.095707 | .407066 |
| .5 | -.024871 | .071414 | -.168381 | 2.3 | .076895 | -.087504 | .484441 |
| .6 | -.035932 | .070698 | -.253435 | 2.4 | .085314 | -.030227 | .586746 |
| .7 | -.046216 | .067923 | -.325120 | 2.5 | .088583 | -.024956 | .530203 |
| .8 | -.054782 | .068691 | -.395488 | 2.6 | .085705 | -.026422 | .514713 |
| .9 | -.061513 | .067923 | -.442312 | 2.7 | .077804 | -.026702 | .472812 |
| 1.0 | -.066176 | .064712 | -.495065 | 2.8 | .066452 | -.019651 | .431104 |
| 1.1 | -.068518 | .043246 | -.529718 | 2.9 | .052350 | -.005864 | .364215 |
| 1.2 | -.068301 | .032077 | -.501830 | 3.0 | .037762 | .000908 | .298661 |
| 1.3 | -.064585 | -.012565 | -.505261 | 3.1 | .023846 | .005620 | .227539 |
| 1.4 | -.056328 | -.022862 | -.425090 | 3.2 | .012066 | .007958 | .109150 |
| 1.5 | -.044150 | -.026023 | -.372095 | 3.3 | .001887 | .010611 | .092732 |
| 1.6 | -.030073 | -.047260 | -.266469 | 3.4 | -.006638 | .009145 | .010140 |
| 1.7 | -.014737 | -.075428 | -.146952 | 3.5 | -.012536 | .010175 | -.049418 |

$$\Delta\psi = C_{L_\alpha}\,\Delta\alpha + C_{L_\delta}\,\Delta\delta$$

$$\begin{bmatrix} 0.087761 & -0.053345 \\ -0.053345 & 0.100964 \end{bmatrix} \begin{Bmatrix} C_{L_\alpha} \\ C_{L_\delta} \end{Bmatrix} = \begin{Bmatrix} 0.598132 \\ -0.332334 \end{Bmatrix}$$

$$C_{L_\alpha} = 7.09$$

$$C_{L_\delta} = 0.456$$

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TABLE V.- COMPUTATIONS ILLUSTRATING METHOD B - Concluded

(c) Final results from three sets of flight data using method B with $\lambda = 0.5$

| | Flight 1 | Probable error for flight 1 | Flight 2 | Flight 3 |
|---|---|---|---|---|
| $C_{L_\alpha}$ | 7.09 | 0.113 | 6.68 | 6.78 |
| $C_{L_\delta}$ | 0.456 | 0.105 | 0.402 | 0.420 |
| $C_{L_{\dot{\theta}}}$ | 0.062 | 0.008 | 0.057 | 0.053 |
| $C_{L_{\dot{\alpha}}}$ | 0.031 | 0.004 | 0.028 | 0.026 |
| $C_{m_\alpha}$ | -0.624 | 0.026 | -0.670 | -0.698 |
| $C_{m_{\dot{\theta}}}$ | -0.149 | 0.019 | -0.136 | -0.126 |
| $C_{m_{\dot{\alpha}}}$ | -0.075 | 0.010 | -0.068 | -0.063 |
| $C_{m_\delta}$ | -0.861 | 0.063 | -0.813 | -0.892 |
| $(\partial C_L/\partial\alpha)_t$ | 4.21 | | 3.79 | 3.52 |
| $\partial C_{L_t}/\partial\delta$ | 2.13 | | 1.87 | 1.96 |

TABLE VI.- COMPUTATIONS ILLUSTRATING METHOD C

(a) Determination of $K_1$, $K_2$, $K_5$, and $K_6$ from flight 1 data

| $t$ | $\Delta\theta$ | $\int_0^t \Delta\theta \, dt$ | $-\int_0^t \Delta\delta \, dt$ | $-\int_0^t \int_0^\tau \Delta\delta \, d\tau \, dt$ | $-\Delta\dot{\theta}$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| .1 | 0 | 0 | -.00016 | 0 | 0 |
| .2 | -.000675 | -.000017 | -.00333 | -.00018 | .019808 |
| .3 | -.005124 | -.000262 | -.00933 | -.00082 | .074064 |
| .4 | -.015308 | -.001241 | -.01734 | -.00217 | .125305 |
| .5 | -.029159 | -.003444 | -.02473 | -.00427 | .149419 |
| .6 | -.044962 | -.007137 | -.03205 | -.00711 | .165781 |
| .7 | -.061712 | -.012469 | -.03921 | -.01068 | .167505 |
| .8 | -.078140 | -.019465 | -.04624 | -.01495 | .163628 |
| .9 | -.094122 | -.028085 | -.05317 | -.01992 | .155447 |
| 1.0 | -.109042 | -.038253 | -.05994 | -.02558 | .142960 |
| 1.1 | -.122585 | -.049848 | -.06560 | -.03188 | .127027 |
| 1.2 | -0.133601 | -0.062688 | -0.06933 | -0.03863 | 0.090426 |
| 1.3 | -.140928 | -.076444 | -.07124 | -.04570 | .053810 |
| 1.4 | -.142456 | -.090671 | -.06946 | -.05274 | -.014640 |
| 1.5 | -.138768 | -.104769 | -.06726 | -.05958 | -.058562 |
| 1.6 | -.131333 | -.118301 | -.06336 | -.06613 | -.091287 |
| 1.7 | -.120101 | -.130907 | -.05716 | -.07218 | -.133055 |
| 1.8 | -.104585 | -.142170 | -.04975 | -.07752 | -.167503 |
| 1.9 | -.086708 | -.151759 | -.04230 | -.08213 | -.196353 |
| 2.0 | -.066656 | -.159437 | -.03387 | -.08594 | -.207549 |
| 2.1 | -.045823 | -.165066 | -.02518 | -.08890 | -.214008 |
| 2.2 | -.024343 | -.168577 | -.01588 | -.09096 | -.217022 |
| 2.3 | -.002525 | -.169910 | -.00649 | -.09207 | -.217022 |
| 2.4 | .018660 | -.169068 | -.00126 | -.09241 | -.186880 |

$$K_1 \Delta\theta + K_2 \int_0^t \Delta\theta \, dt - K_5 \int_0^t \Delta\delta \, dt - K_6 \int_0^t \int_0^\tau \Delta\delta \, d\tau \, dt = -\Delta\dot{\theta}$$

$$\begin{bmatrix} 0.183701353 & 0.141082469 & 0.094470059 & 0.081352276 \\ 0.141082469 & 0.247220311 & 0.073939691 & 0.136293337 \\ 0.094470059 & 0.073939691 & 0.049289973 & 0.042731928 \\ 0.081352276 & 0.136293337 & 0.042731928 & 0.075308217 \end{bmatrix} \begin{Bmatrix} K_1 \\ K_2 \\ K_5 \\ K_6 \end{Bmatrix} = \begin{Bmatrix} -0.004034232 \\ 0.229668158 \\ -0.008849134 \\ 0.119729023 \end{Bmatrix}$$

$K_1 = 4.14$

$K_2 = 9.55$

$K_5 = -9.77$

$K_6 = -14.62$

TABLE VI.- COMPUTATIONS ILLUSTRATING METHOD C - Continued

(b) Determination of $K_1$, $K_2$, $K_5$, and $K_6$ from flight 2 data

| $t$ | $\Delta\theta$ | $\int_0^t \Delta\theta\, dt$ | $-\int_0^t \Delta\delta\, dt$ | $-\int_0^t \int_0^\tau \Delta\delta\, d\tau\, dt$ | $-\Delta\dot{\theta}$ | $t$ | $\Delta\theta$ | $\int_0^t \Delta\theta\, dt$ | $-\int_0^t \Delta\delta\, dt$ | $-\int_0^t \int_0^\tau \Delta\delta\, d\tau\, dt$ | $-\Delta\dot{\theta}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 1.3 | -0.148557 | -0.081124 | -0.075188 | -0.048259 | 0.032391 |
| .1 | 0 | 0 | .000094 | .000025 | 0 | 1.4 | -.146921 | -.095959 | -.070181 | -.055559 | -.053825 |
| .2 | -.000377 | -.000008 | -.002278 | -.000063 | .013779 | 1.5 | -.138050 | -.110267 | -.062688 | -.062211 | -.122290 |
| .3 | -.004005 | -.000200 | -.008715 | -.000599 | .065021 | 1.6 | -.122660 | -.123344 | -.054155 | -.068062 | -.182144 |
| .4 | -.013776 | -.001049 | -.016785 | -.001861 | .126166 | 1.7 | -.102296 | -.134634 | -.045741 | -.073056 | -.221759 |
| .5 | -.028391 | -.003117 | -.025055 | -.003952 | .161906 | 1.8 | -.079073 | -.143710 | -.037416 | -.077213 | -.239844 |
| .6 | -.045744 | -.006811 | -.033384 | -.006874 | .183005 | 1.9 | -.054902 | -.150417 | -.028839 | -.080588 | -.240705 |
| .7 | -.064561 | -.012315 | -.041523 | -.010622 | .191186 | 2.0 | -.031272 | -.154720 | -.020024 | -.082973 | -.232924 |
| .8 | -.083762 | -.019738 | -.049313 | -.015167 | .190325 | 2.1 | -.008268 | -.156692 | -.010793 | -.084517 | -.228218 |
| .9 | -.102124 | -.029039 | -.056182 | -.020438 | .174393 | 2.2 | .014414 | -.156370 | -.001206 | -.085120 | -.221759 |
| 1.0 | -.118566 | -.040094 | -.063462 | -.026417 | .153294 | 2.3 | .035352 | -.153867 | .006292 | -.084852 | -.193339 |
| 1.1 | -.132553 | -.052670 | -.071755 | -.033208 | .125305 | 2.4 | .052386 | -.149447 | .012185 | -.083915 | -.148988 |
| 1.2 | -.143128 | -.066497 | -.076422 | -.040647 | .083106 | 2.5 | .065480 | -.143521 | .018677 | -.082379 | -.114540 |

$$K_1 \Delta\theta + K_2 \int_0^t \Delta\theta\, dt - K_5 \int_0^t \Delta\delta\, dt - K_6 \int_0^t \int_0^\tau \Delta\delta\, d\tau\, dt = -\Delta\dot{\theta}$$

$$\begin{bmatrix} 0.18385708 & 0.09819197 & 0.09180851 & 0.05739522 \\ 0.09819197 & 0.25436118 & 0.05665420 & 0.14095293 \\ 0.09180851 & 0.05665420 & 0.04725485 & 0.03311576 \\ 0.05739522 & 0.14095293 & 0.03311576 & 0.07828319 \end{bmatrix} \begin{Bmatrix} K_1 \\ K_2 \\ K_5 \\ K_6 \end{Bmatrix} = \begin{Bmatrix} -0.02525329 \\ 0.28335143 \\ -0.01717559 \\ 0.14957619 \end{Bmatrix}$$

$K_1 = 4.19$

$K_2 = 10.33$

$K_5 = -10.01$

$K_6 = -15.53$

TABLE VI.- COMPUTATIONS ILLUSTRATING METHOD C - Concluded

(c) Final results using method C with $\lambda = 0.5$

| | Flight 1 | | | Flight 2 | |
|---|---|---|---|---|---|
| | Accurate values | Approximate values | Probable error | Accurate values | Approximate values |
| $C_{L_\alpha}$ | 7.21 | 7.00 | 0.106 | 7.59 | 7.34 |
| $C_{L_\delta}$ | 0.374 | 0.371 | 0.013 | 0.394 | 0.391 |
| $C_{L_{\dot{\theta}}}$ | 0.066 | 0.067 | 0.005 | 0.067 | 0.069 |
| $C_{L_{\dot{\alpha}}}$ | 0.033 | 0.034 | 0.003 | 0.034 | 0.034 |
| $C_{m_\alpha}$ | -0.624 | -0.627 | 0.073 | -0.706 | -0.710 |
| $C_{m_{\dot{\theta}}}$ | -0.158 | -0.160 | 0.012 | -0.161 | -0.164 |
| $C_{m_{\dot{\alpha}}}$ | -0.079 | -0.080 | 0.006 | -0.081 | -0.082 |
| $C_{m_\delta}$ | -0.894 | -0.887 | 0.032 | -0.941 | -0.935 |
| $(\partial C_L/\partial\alpha)_t$ | 4.46 | 4.54 | | 4.49 | 4.58 |
| $\partial C_{L_t}/\partial\delta$ | 1.75 | 1.73 | | 1.84 | 1.83 |

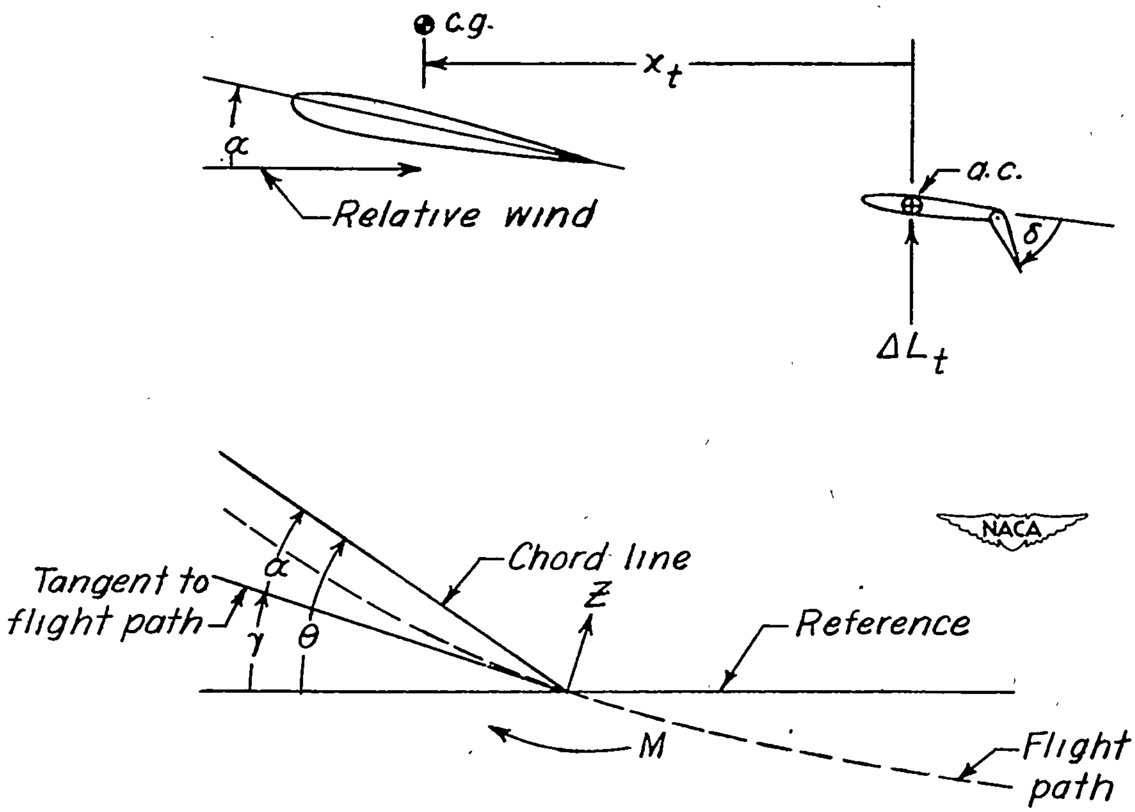


Figure 1.- Sign conventions employed. Positive directions shown.

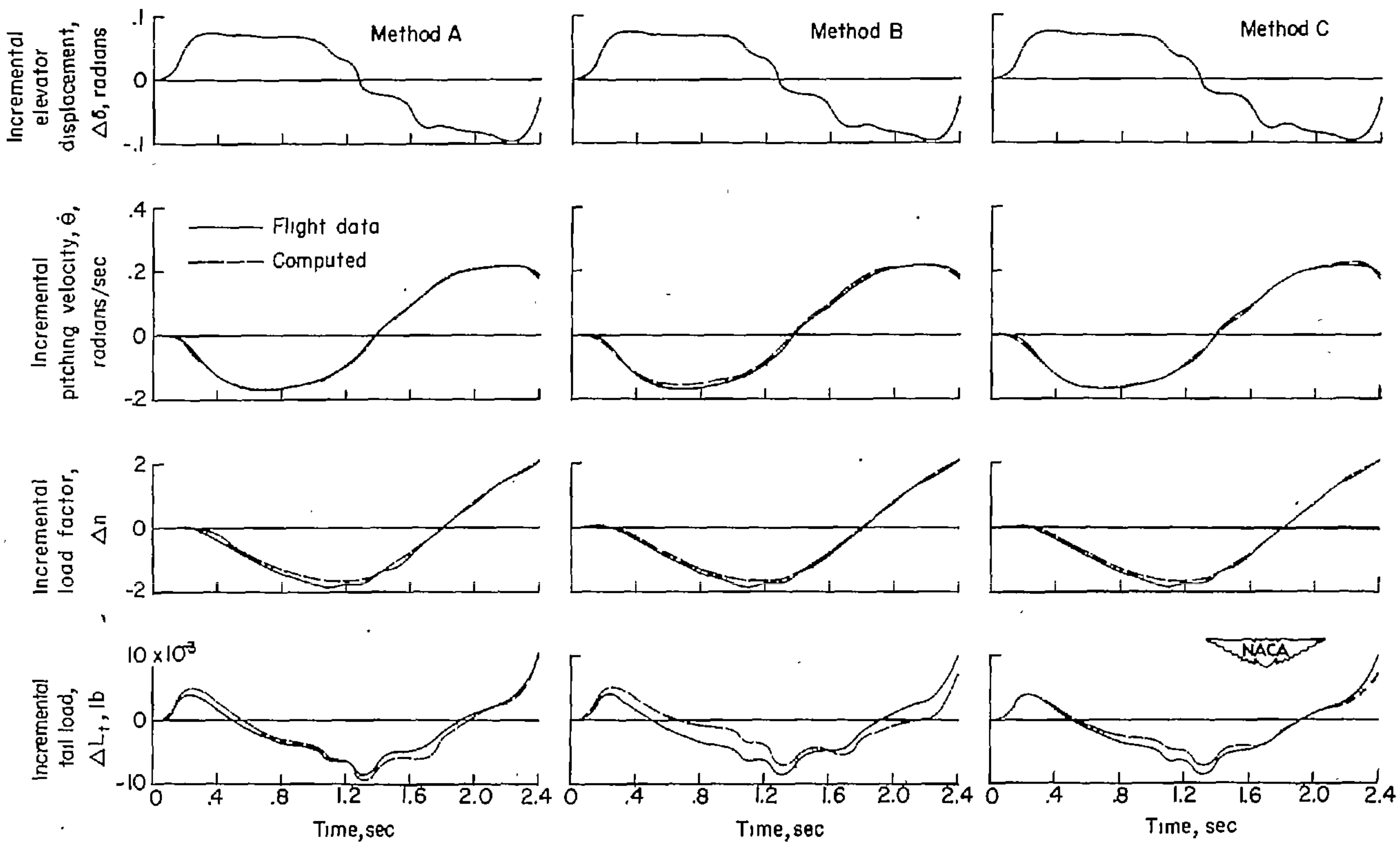


Figure 2.- Measured and computed flight 1 time histories of incremental elevator displacement, pitching velocity, load factor, and tail load showing the fit of the data.



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